TUTORIALS FOR
Design of Experiments Analyses

STATGRAPHICS *PLUS*® FOR WINDOWS

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Introduction

This manual contains tutorials for the Design of Experiments analyses in STATGRAPHICS *Plus*.

For information about design of experiments in general, see the following sections in the online help system, *Using Experimental Designs, Basic Principles of Experimental Design,* and *Understanding Design Classes*.

Tutorials in This Manual

The tutorials in this manual are:

- *Creating and Analyzing a Screening Design*
- *Creating and Analyzing a Response Surface Design*
- *Creating and Analyzing a Mixture Design*
- *Augmenting a Design*

TUTORIAL 1

Creating and Analyzing a Screening Design

In this tutorial, which is adapted from an experiment by Vardeman (1994) in *Statistics for Engineering Problem Solving,* you will study flight distances of paper airplanes.

Four primary factors are thought to influence the distance the planes fly when launched from a prefabricated launcher:

- design of the plane (straight or tee)
- use of a paper clip on the nose (yes or no)
- type of paper (construction or notebook)
- type of wing tips (straight or bent).

To duplicate this experiment, you will use a 2^4 factorial design (four factors, each having two different levels). The response variable is **Distance**, which represents the mean flight distance for two launches of each plane.

To begin, open STATGRAPHICS *Plus* and the **Plane** data file.

Creating the Design

1. Choose SPECIAL... EXPERIMENTAL DESIGN... CREATE DESIGN... from the Menu bar to display the Create Design Options dialog box.

2. Accept Screening as the Design Class default.

3. Accept the default, 1, in the No. of Response Variables text box.

4. Type ***4*** in the Number of Experimental Factors text box.

5. Type ***Airplane Experiment*** in the Comment text box.

This name will appear on reports and graphs when you analyze the experiment. Your dialog box should now look like the one shown in Figure 1-1.

6. Click OK to display the Factor Definition Options dialog box.

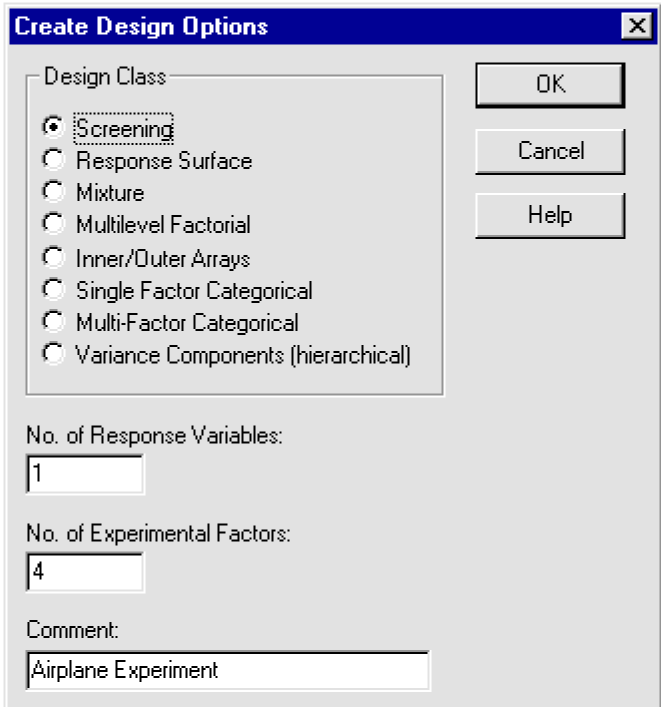


Figure 1-1. Completed Create Design Options Dialog Box (Screening Design)

7. Complete the dialog box for Factor A: Type ***Design*** in the Name text box, ***Straight*** in the Low text box, ***Tee*** in the High text box, leave the Units text box blank, and the Continuous check box deselected (see Figure 1-2).

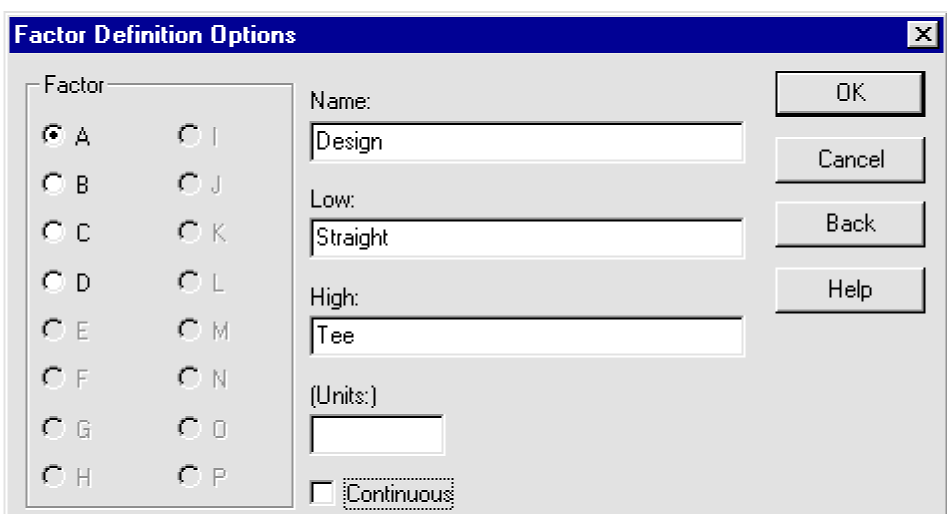


Figure 1-2. Completed Dialog Box for Factor A

8. Complete the dialog box for Factor B: Click the Factor B option, type ***Nose*** in the Name text box, ***None*** in the Low text box, ***Clip*** in the High text box, leave the Units text box blank, and the Continuous check box deselected (see Figure 1-3).

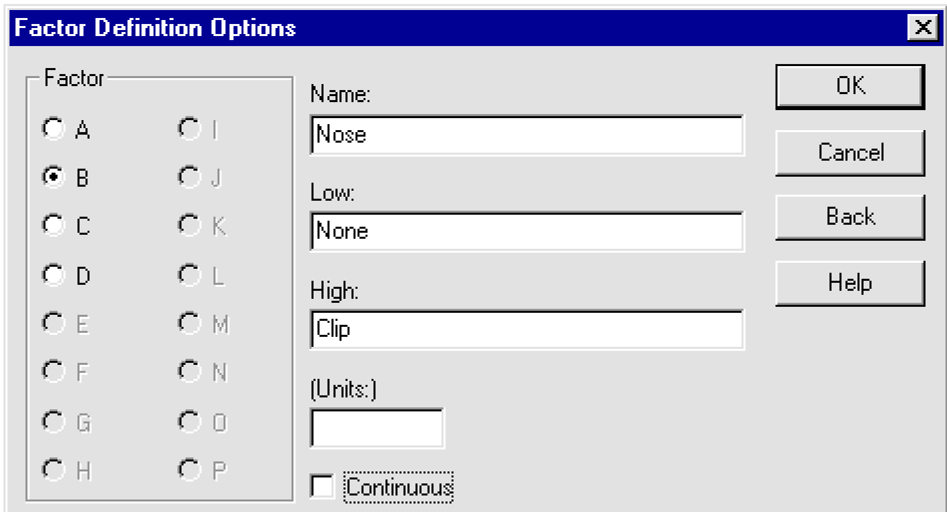


Figure 1-3. Completed Dialog Box for Factor B

9. Complete the dialog box for Factor C: Click the Factor Coption, type ***Paper*** in the Name text box, ***Notebook*** in the Low text box, ***Construct*** in the High text box, leave the Units text box blank, and the Continuous check box deselected (see Figure 1-4).

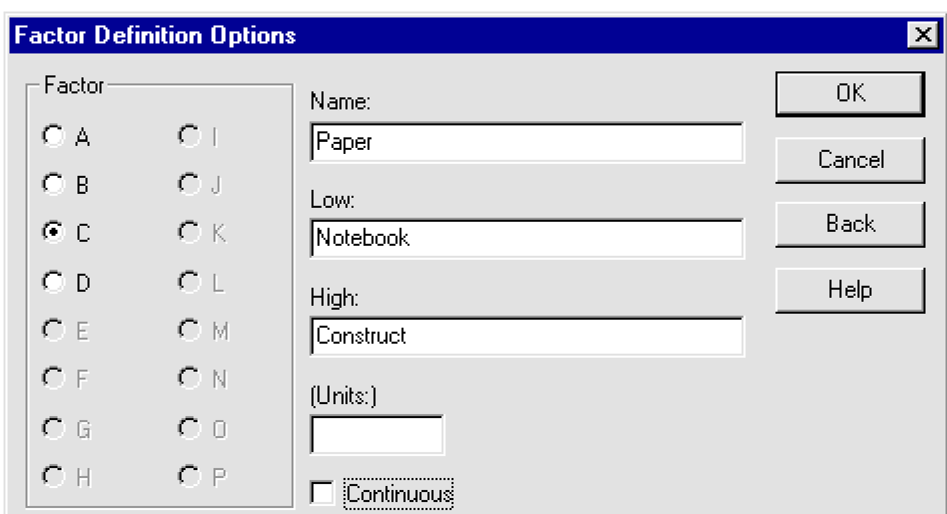


Figure 1-4. Completed Dialog Box for Factor C

10. Complete the dialog box for Factor D: Click the Factor D option, type ***Wing*** in the Name text box, ***Straight*** in the Low text box, ***Bent*** in the High text box, leave the Units text box blank, and the Continuous check box deselected (see Figure 1-5).

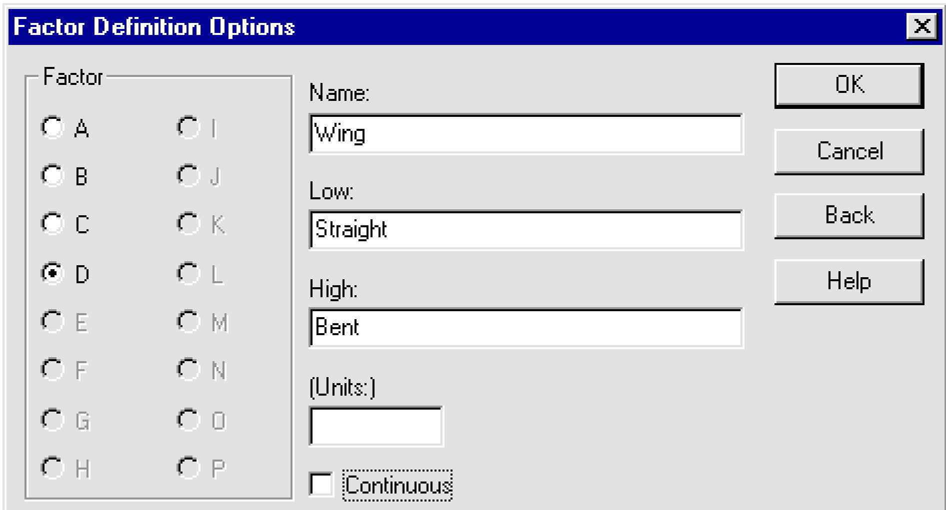


Figure 1-5. Completed Dialog Box for Factor D

11. Click OK to display the Response Definition Options dialog box.

12. Type ***Distance*** in the Name text box.

13. Type ***Feet*** in the Units text box.

The dialog box should look like the one shown in Figure 1-6.

14. Click OK to display the Screening Design Selection dialog box.

15. Use the down arrow to display the available designs, then choose ***Factorial in 2 blocks*** (see Figure 1-7).

16. Click OK to display the Blocked Screening Design Options dialog box.

Notice that the name of the base design (Factorial 2^4) appears on the first line of the dialog box. Beneath that, the number of runs and error degrees of freedom are shown.

17. Deselect the Randomize check box so the design will be created in standardized order to make it easier to enter data then accept the other defaults (see Figure 1-8).

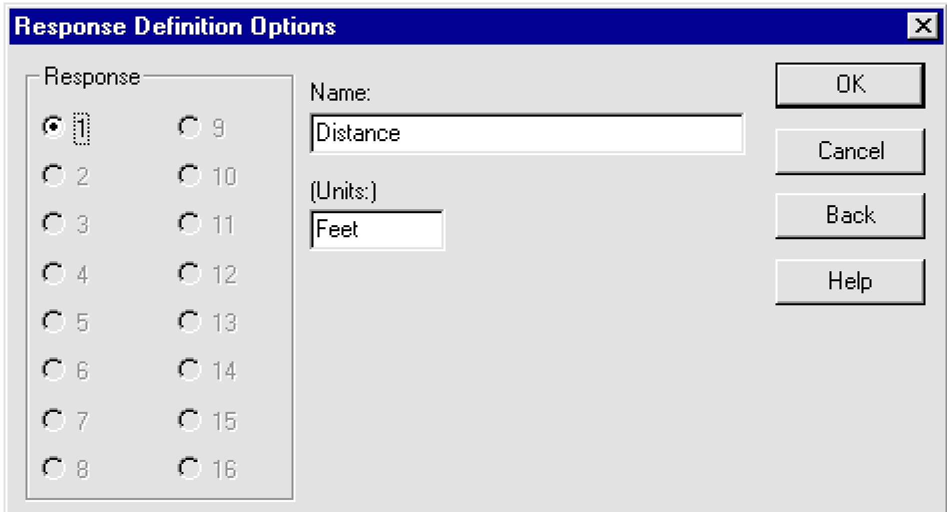


Figure 1-6. Completed Response Definition Options Dialog Box

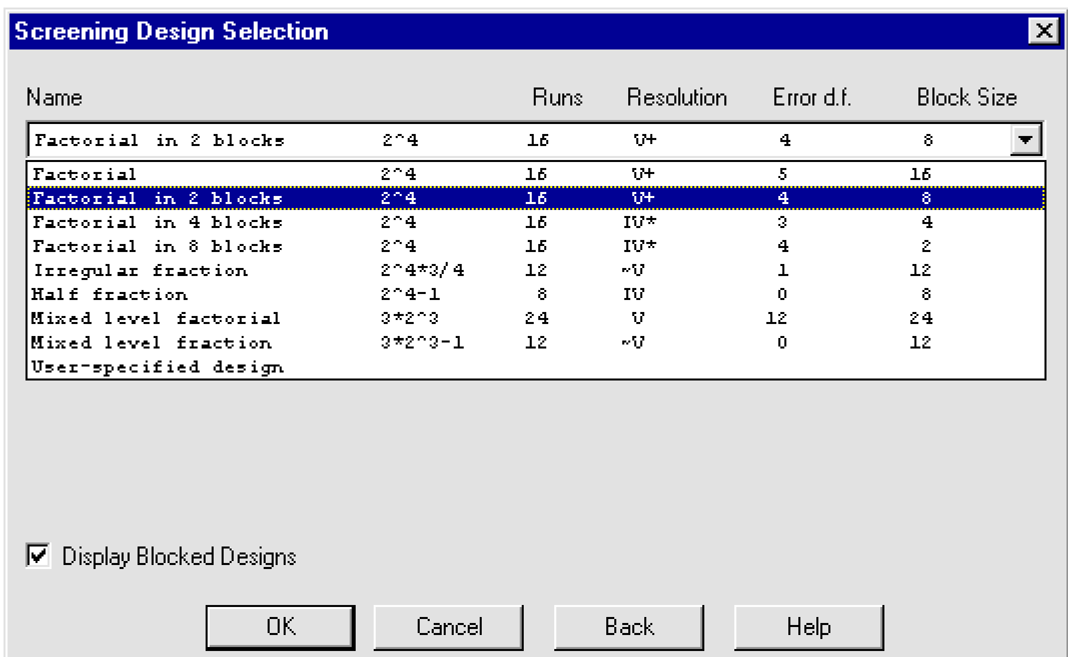


Figure 1-7. Screening Design Selection Dialog Box with Chosen Design

Note: In a typical experiment, you would normally randomize the design.

18. Click OK to display the Design Summary in the Screening Design Attributes window (see Figure 1-9).

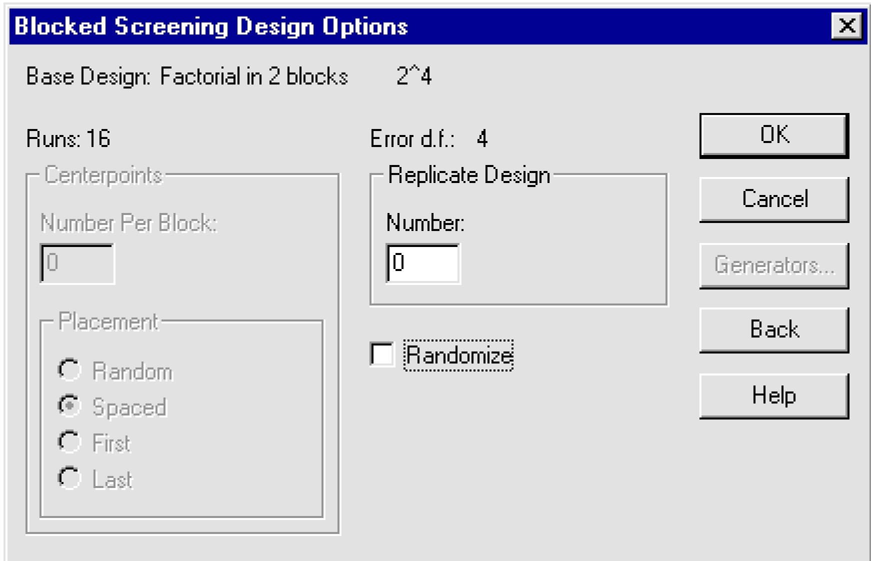


Figure 1-8. Completed Blocked Screening Design Options Dialog Box

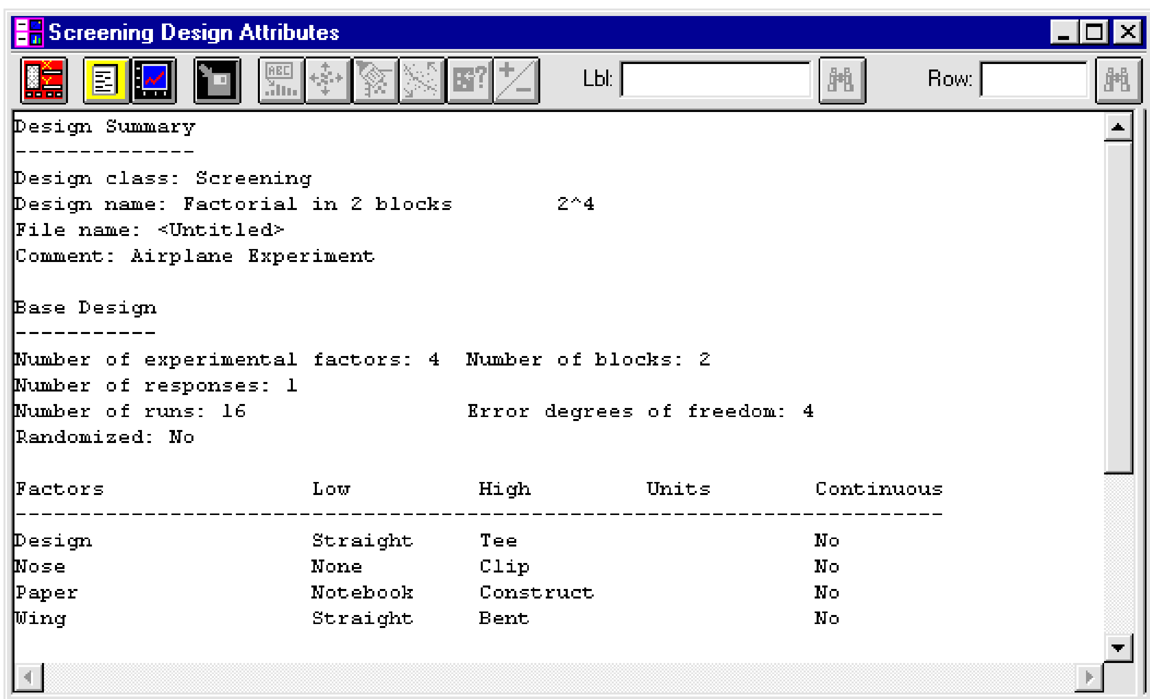


Figure 1-9. Design Summary

The Design Summary includes the name of the design and its class, and the comment you entered about the experiment. If you have not yet saved the design, the file name will appear as <Untitled>. The rest of the information summarizes the factors, responses, runs, blocks, number of centerpoints, and error degrees of freedom.

After reviewing the details of the design, save it.

Naming and Saving the Design

1. Choose FILE... SAVE AS... SAVE DESIGN FILE AS... from the Menu bar to display the Save Design File As dialog box.

2. Type ***Plane.sfx*** in the Name text box and click the Save button to name and save the file and redisplay the Design Summary.

Now you are ready to collect and set up the data for the experiment. The design and response values are shown in Table 1-1.

Table 1-1. The Design and Response Values for the Plane.sfx

Design	*Nose*	*Paper*	*Wing*
Tee	None	Notebook	Straight
Straight	Clip	Notebook	Straight
Straight	None	Construct	Straight
Tee	Clip	Construct	Straight
Straight	None	Notebook	Bent
Tee	Clip	Notebook	Bent
Tee	None	Construct	Bent
Straight	Clip	Construct	Bent
Straight	None	Notebook	Straight
Tee	Clip	Notebook	Straight
Tee	None	Construct	Straight
Straight	Clip	Construct	Straight
Tee	None	Notebook	Bent
Straight	Clip	Notebook	Bent
Straight	None	Construct	Bent
Tee	Clip	Construct	Bent

Entering and Saving the Data

1. Open a new DataSheet and enter the information from Table 1-1 into it, using the headings of the table for the names of the columns on the DataSheet.

2. Save the data file as a design file, using FILE... SAVE AS... SAVE DESIGN FILE AS... , and ***Plane.sfx*** as the design name.

Now you are ready to analyze the design.

Analyzing the Experimental Results

1. Choose SPECIAL... EXPERIMENTAL DESIGN... ANALYZE DESIGN... from the Menu bar to display the Analyze Design dialog box.

2. Enter Distance into the Data text box and click OK to display the Analysis Summary and the Standardized Pareto Chart in the Analysis window. Maximize the Analysis Summary (see Figure 1-11).

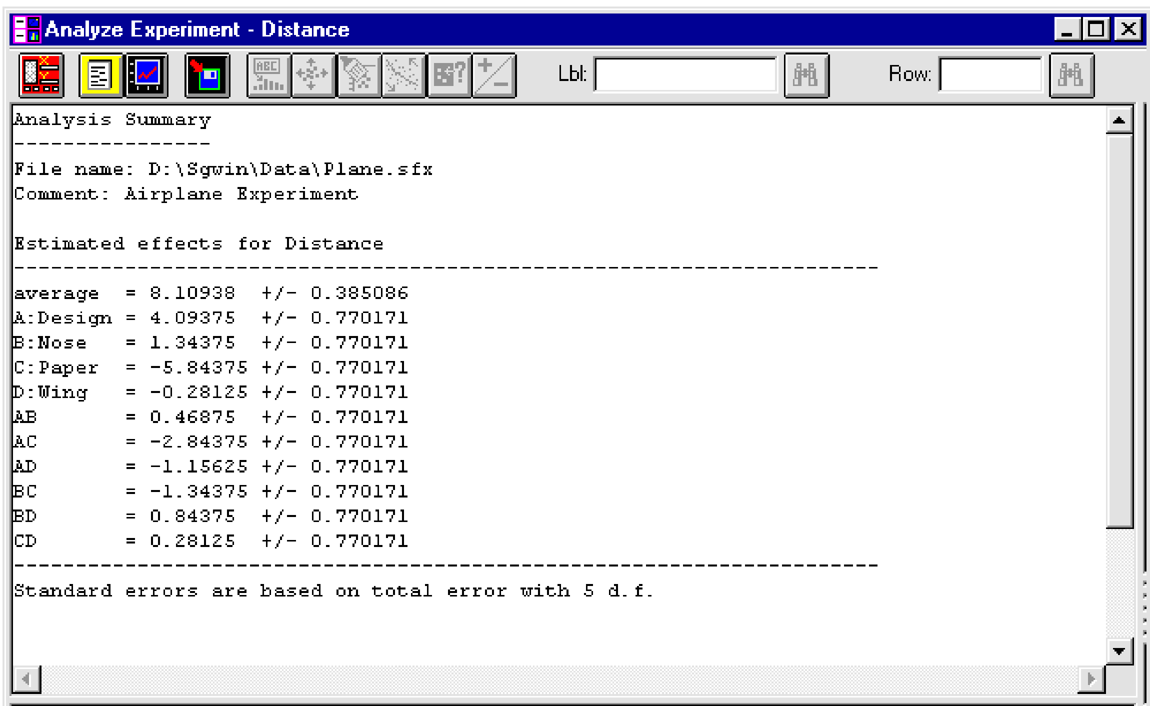


Figure 1-11. The Analysis Summary for the Airplane Experiment Design

Because you chose a full factorial design and collected 16 runs, all the main and two-factor interaction effects are estimates. You can ignore interactions greater than order 2 because higher-order interactions are typically negligible. To determine which factors contribute significantly to the flight distance, display the Pareto Chart.

3. Minimize the Analysis Summary and maximize the Pareto Chart (see Figure 1-12).

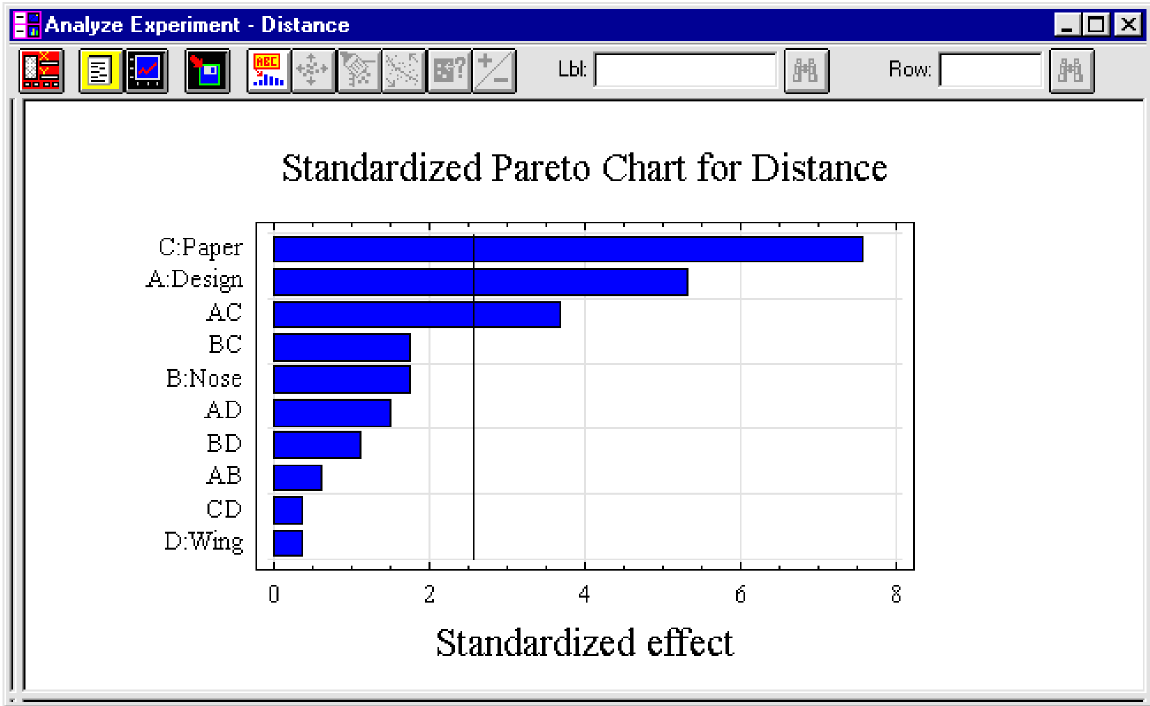


Figure 1-12. Standardized Pareto Chart

The chart shows that Paper (C), Design (A), and the AC interaction are significant effects because they cross the vertical line that represents the 95 percent test for significance.

The Main Effects and Normal Probability Plots of Effects could also substantiate these findings.

4. Click the Graphical Options button to display the dialog box, then click the Main Effects Plots, Interaction Plots, and Normal Probability Plots of Effects check boxes.

5. Click OK to display the plots in the second, third, and fourth graphics panes of the Analysis window.

6. Maximize the Normal Probability Plot of Effects (the fourth graphics pane).

7. Click the right mouse button on the plot, then the left on Pane Options to display the Effects Normal Probability Plot Options dialog box.

8. Choose the Half-Normal Plot, accept the defaults, Vertical, in the Direction section and Fitted Line, then click Label Effects. The dialog box should look like the one shown in Figure 1-13.

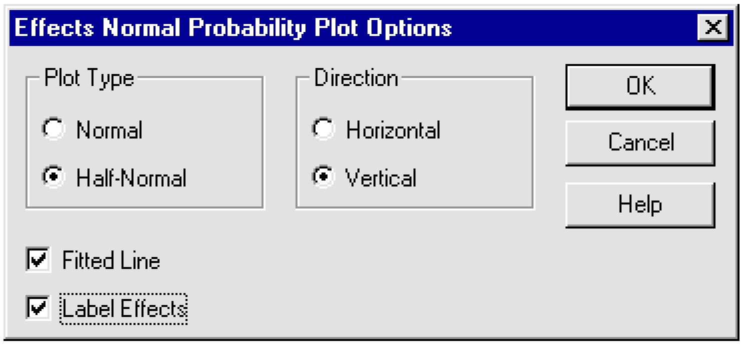


Figure 1-13. Completed Options Dialog Box

9. Click OK to display the Half-Normal Plot (see Figure 1-14).

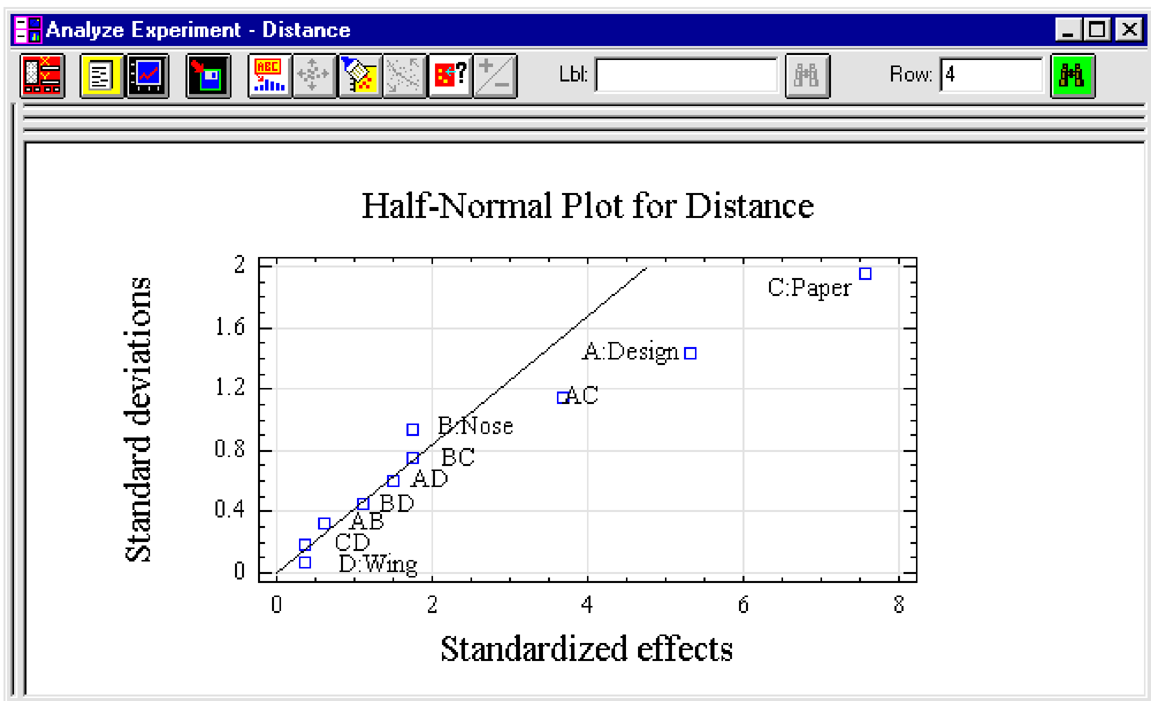


Figure 1-14. Half-Normal Plot

The plot shows that Paper, Design, and the AC interaction all fall to the right of the vertical line, which indicates that they are large effects compared with the other effects in the model.

10. Minimize the Half-Normal Plot, and maximize the Interaction Plot (the third graphics pane).

11. Click the right mouse button on the plot, then the left on Pane Options to display the Interaction Plot Options dialog box.

12. Click the Design and Paper check boxes, then deselect Nose and Wing so the dialog box looks like that shown in Figure 1-15.

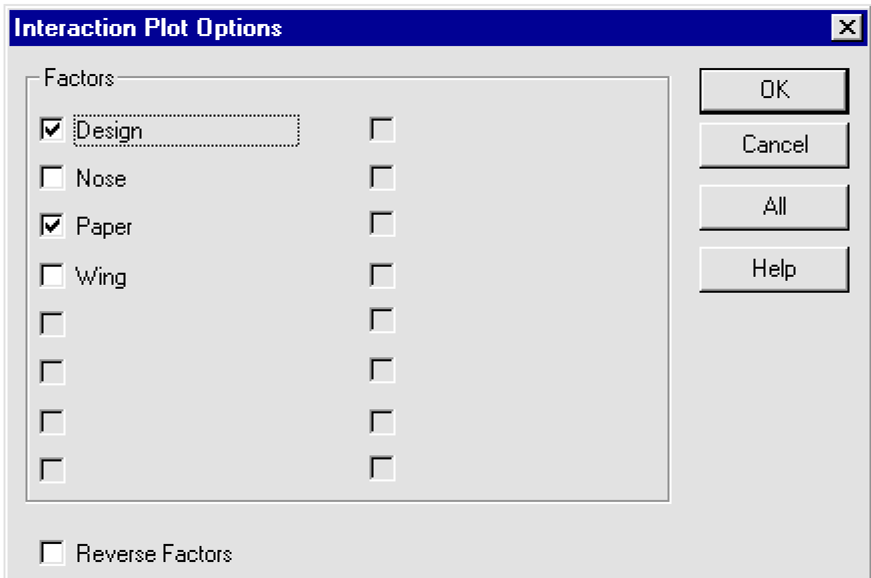


Figure 1-15. Completed Interaction Plot Options Dialog Box

13. Click OK to redisplay the Interaction Plot (see Figure 1-16).

The Interaction Plot shows that the distance is longer for the tee-shaped planes made from notebook paper. An analysis of variance (ANOVA) should also verify these findings.

14. Click the Tabular Options button to display the dialog box, then click the ANOVA Table option, and click OK to display the table. Maximize the table (see Figure 1-17).

The ANOVA Table shows that Factors A, C, and the interaction, AC are statistically significant (p-value < .05). Factors B and D and all the interactions among these factors are negligible.

Based on these findings, it seems that the type of wing and the paper type most significantly influence flight distance. The Main Effects and Interaction Plots show that tee-shaped planes made of notebook paper result in longer flight distances.

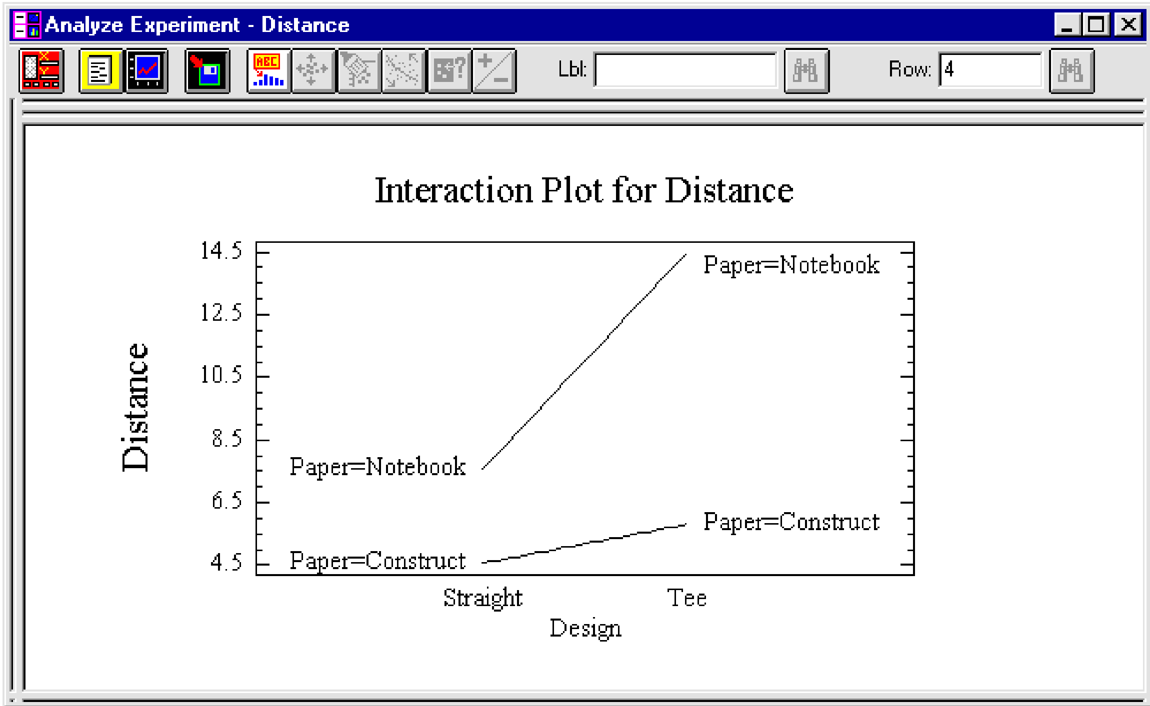


Figure 1-16. The Interaction Plot for the Airplane Distance Experiment

Analyze Experiment - Distance

Analysis of Variance for Distance - Airplane Experiment

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
A:Design	67.0352	1	67.0352	28.25	0.0032
B:Nose	7.22266	1	7.22266	3.04	0.1415
C:Paper	136.598	1	136.598	57.57	0.0006
D:Wing	0.316406	1	0.316406	0.13	0.7299
AB	0.878906	1	0.878906	0.37	0.5694
AC	32.3477	1	32.3477	13.63	0.0141
AD	5.34766	1	5.34766	2.25	0.1936
BC	7.22266	1	7.22266	3.04	0.1415
BD	2.84766	1	2.84766	1.20	0.3232
CD	0.316406	1	0.316406	0.13	0.7299
Total error	11.8633	5	2.37266		
Total (corr.)	271.996	15			

R-squared = 95.6384 percent
R-squared (adjusted for d.f.) = 86.9153 percent

Figure 1-17. The ANOVA Table

References

Vardeman, S. T. 1994. *Statistics for Engineering Problem Solving*. Boston: PWS Publishing Co.

TUTORIAL 2

Creating and Analyzing a Response Surface Design

This example is adapted from Cornell (1990), *How to Apply Response Surface Methodology*. In it you will create and analyze an experimental design undertaken by an engineer who worked for a plant that manufactured plastics.

To produce disks of a uniform width, the engineer needed to minimize the amount of disk wear. He decided to investigate the factors that influenced disk wear during production. Because disk wear was an inherent outcome of production, he consulted with production engineers to come up with a list of possible factors. Jointly, the engineers decided to investigate two factors that they believed most greatly affected disk wear, the

- composition of the disk, which was a ratio of filler to epoxy resin, and
- position of the disk in the mold.

They also decided that the response variable would be the thickness of the disk after subjecting it to an abrasion test.

Initially, they chose a replicated 2^2 factorial design to determine if a first-order model was sufficient. After running this design, they determined that the first-order model fit the response surface in the immediate region. But, because the ultimate goal of the experiment was to map the response surface and determine which combination of factor values would produce the minimum amount of wear, they decided to continue experimentation along the path of steepest ascent.

To do this, they performed a second 2^2 design along the path of steepest ascent, which proved that the first-order model was no longer adequate. To fit the second-order model, they chose a two-factor, central composite design.

You will begin this tutorial by creating a two-factor, central composite design, which is appropriate for more carefully studying an experimental design than a single factorial design. The reason is that a central composite design has two parts: a cube and a star. The cube corresponds to a factorial screening design. For example, in a two-factor study, four points in a central composite design form a cube, which corresponds to a 2^2 factorial study. The star consists of an additional set of points arranged at equal distances from the center of the cube on radii that pass through the center of each face of the cube. The distance from the center of cube to one of these points is known as the *"axial distance"* of the star.

Orthogonality and rotatability are two other desirable attributes of central composite designs. You want a rotatable design; that is, you want a design in which the variance of the predicted response is constant at all points that are equal from the center.

The factors in this study are Factor A, **Risk** (Disk Composition Ratio), and Factor B, **Mold** (Position of Disk in Mold). The response variable is **Thickness**.

To begin, open STATGRAPHICS *Plus*.

Creating the Design

1. Choose SPECIAL... EXPERIMENTAL DESIGN... CREATE DESIGN... from the Menu bar to display the Create Design Options dialog box.

2. Click the Response Surface button under the Design Class options.

3. Accept the default in the Number of Response Variables text box.

4. Type ***2*** in the Number of Experimental Factors text box.

5. Type ***Disk Wear Experiment*** in the Comment text box.

This name will appear on tables when you analyze the experiment. The dialog box should look like the one shown in Figure 2-1.

6. Click OK to display the Factor Definition Options dialog box.

7. Complete the dialog box for Factor A: Type ***Ratio*** in the Name text box, ***.75*** in the Low text box, ***.9*** in the High text box, and leave the Units text box blank (see Figure 2-2).

8. Complete the dialog box for Factor B: Click the Factor B option, then type ***Mold*** in the Name text box, ***.05*** in the Low text box, ***1.0*** in the High text box, and leave the Units text box blank (see Figure 2-3).

9. Click OK to display the Response Definition Options dialog box.

10. Type ***Thickness*** in the Name text box; leave the Units text box blank (see Figure 2-4).

11. Click OK to display the Response Surface Design Selection dialog box.

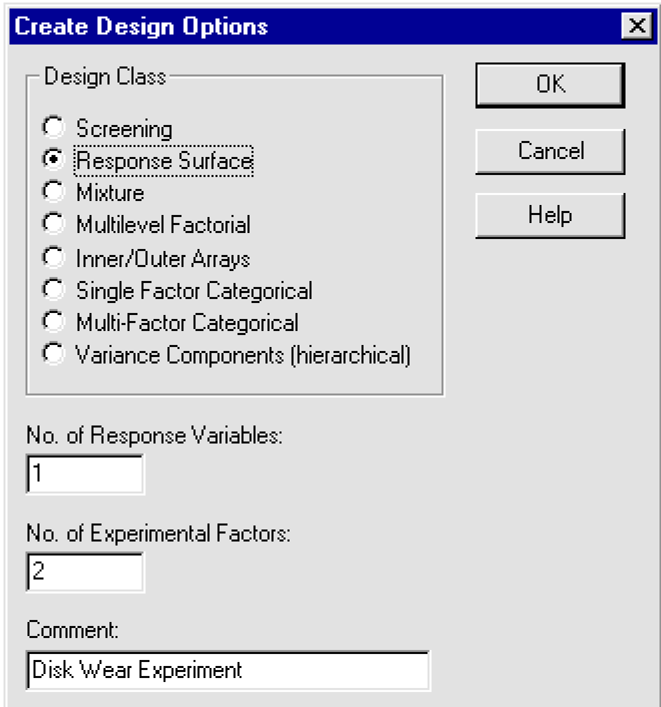


Figure 2-1. Completed Create Design Options Dialog Box

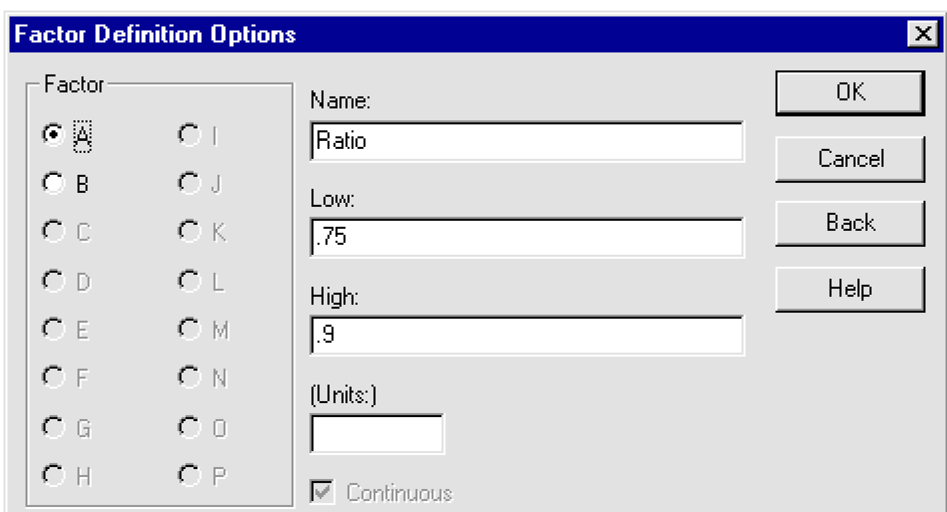


Figure 2-2. Completed Dialog Box for Factor A

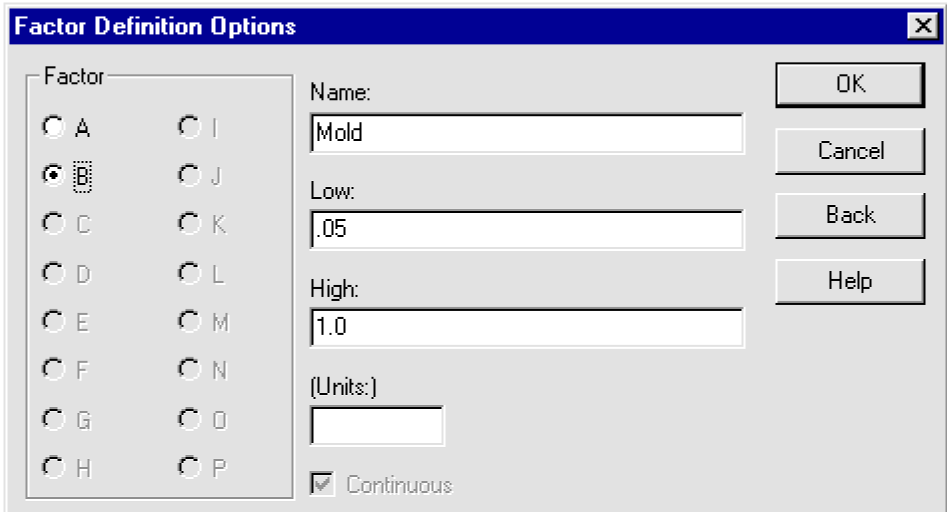


Figure 2-3. Completed Dialog Box for Factor B

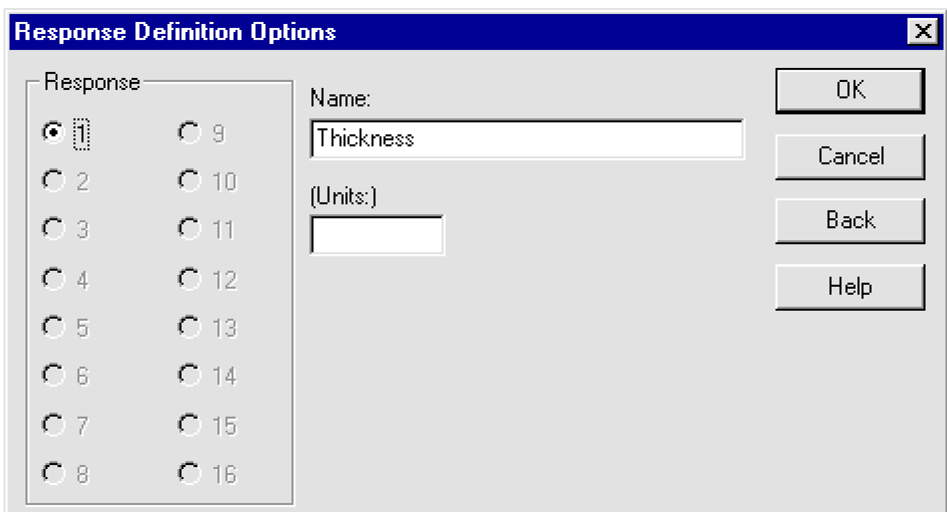


Figure 2-4. Completed Response Definition Options Dialog Box

12. Use the down arrow to display the available designs, then choose Central Composite Design: 2^2 + star (see Figure 2-5).

Notice that the name of the base design (Central composite design: 2^2 + star) appears in the first line of the dialog box. Beneath that, the number of runs and error degrees of freedom are shown.

13. Click OK to display the Composite Design Options dialog box.

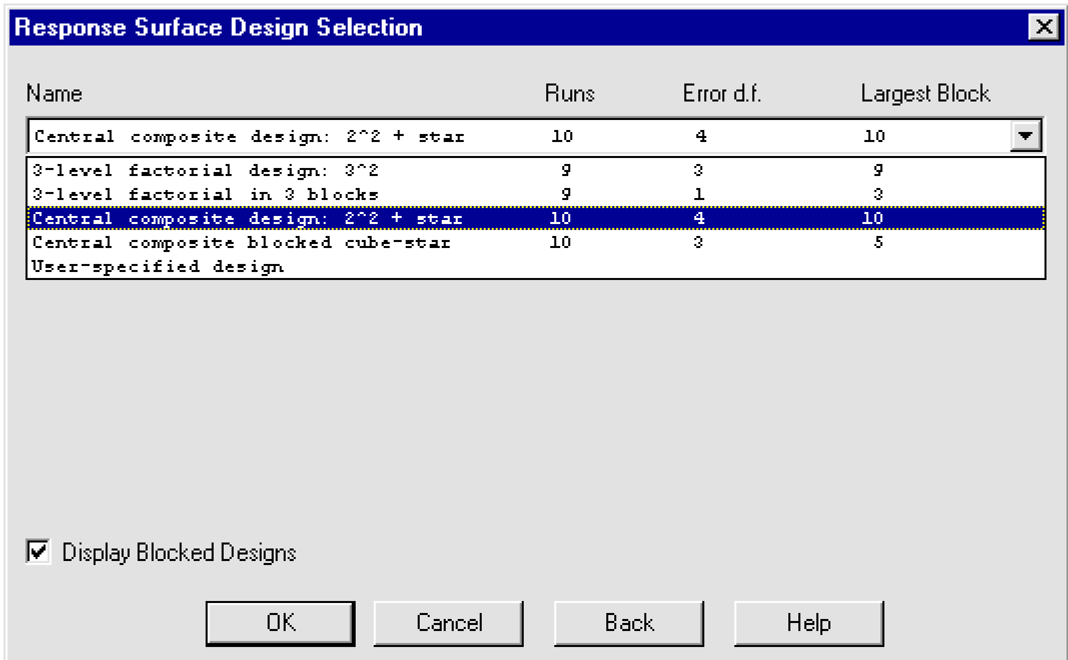


Figure 2-5. Response Surface Design Selection Dialog Box with Chosen Design

14. Click the Response Surface button under the Design Class options, then accept the Rotatable option under the Design Characteristics portion of the dialog box.

15. Accept the default, 2, as the Number of Centerpoints.

16. Change the placement of the centerpoints to **Last**.

17. Accept the defaults for the Number of Replicates and deselect the Randomize check box. The dialog box should look like the one shown in Figure 2-6.

18. Click OK to display the Design Summary in the Response Surface Design Attributes window (see Figure 2-7).

The Design Summary includes the name of the design, its class, and the comment you entered about the experiment. If you have not yet saved the design, the file name will appear as <Untitled>. The rest of the information summarizes the factors, responses, runs, blocks, number of centerpoints, and error degrees of freedom.

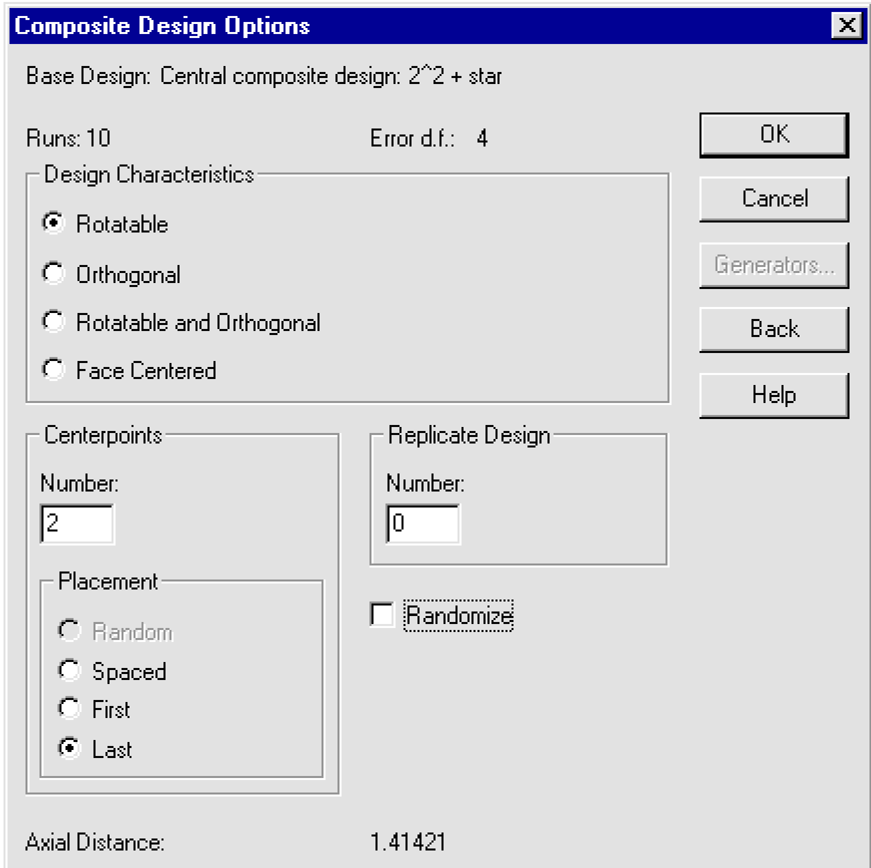


Figure 2-6. Completed Composite Design Options Dialog Box

Naming and Saving the Design

1. Choose FILE... SAVE AS... SAVE DESIGN FILE AS... from the Menu bar to display the Save Design File As dialog box.

2. Type ***Disk.sfx*** in the File Name text box and click the Save button to name and save the file and redisplay the Design Summary.

Now you are ready to collect and set up the data for the experiment. You will print a Worksheet, which will enable you to record the values for the data.

Printing a Worksheet

1. Click the Tabular Options button to display the dialog box, then click the Worksheet option, and OK to display the worksheet in the second pane.

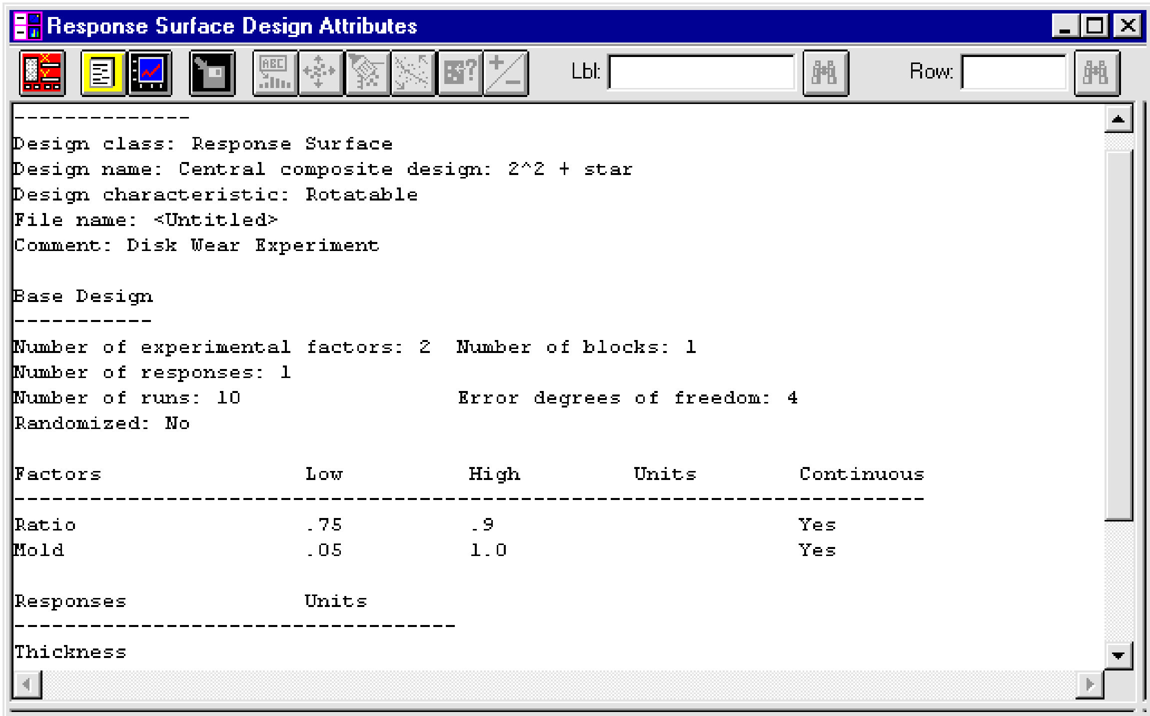


Figure 2-7. Design Summary

2. Choose FILE... PRINT... from the Menu bar to display the Print Analysis dialog box.

3. Choose the All Text Panes option, then click OK to print the Worksheet (see Figure 2-8).

You will find the Worksheet handy when you collect and set up data because it shows the order in which you should conduct the experimental runs and enter responses into the DataSheet.

You are ready to complete the Worksheet. The design and response variables are shown in Table 2-1.

Entering and Saving the Data

1. Open the DataSheet (**Disk.sfx**) and enter the information from Table 2-1 into it.

2. Save the data file as a design file, using FILE... SAVE AS... SAVE DESIGN FILE AS... and ***Disk.sfx*** as the design name.

Now you are ready to analyze the design.

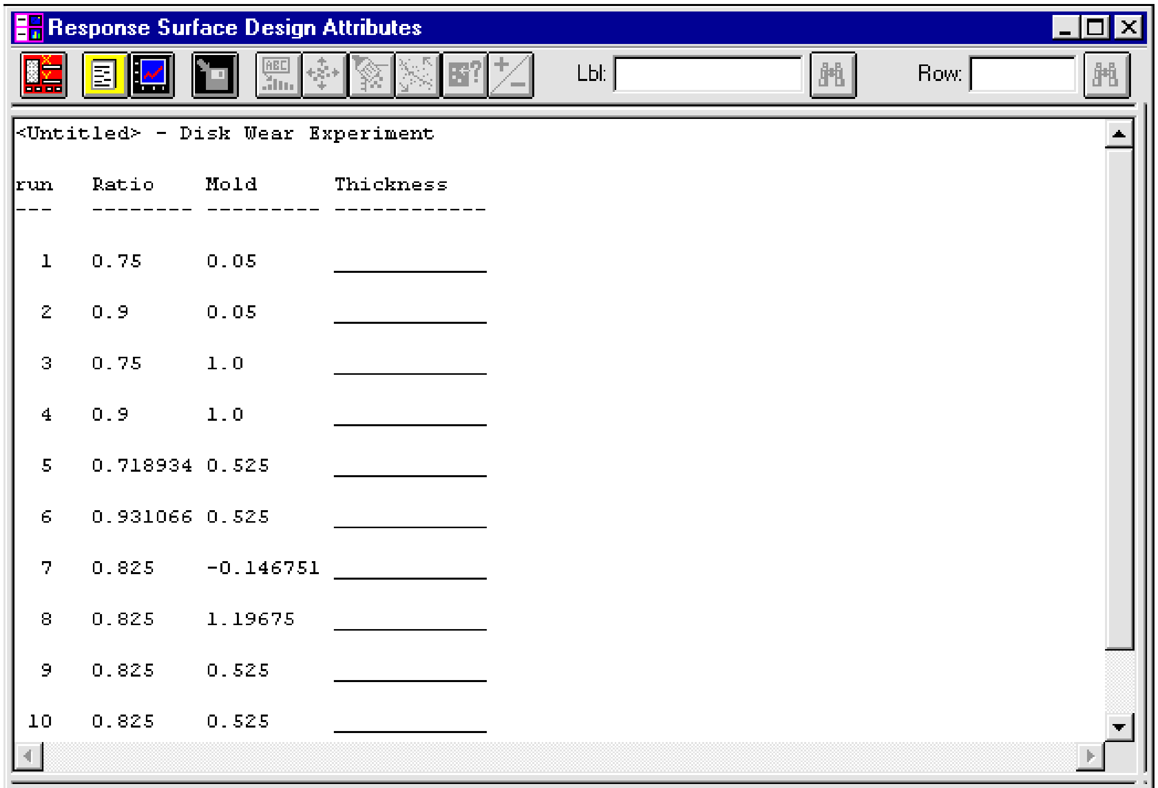


Figure 2-8. Worksheet for the Design

Table 2-1. Design and Response Values for the Disk.sfx File

*Ratio**	*Mold**	*Thickness**
0.75	0.5	7.3
0.9	0.5	7.0
0.75	1.0	7.1
0.9	1.0	8.0
0.718934	0.75	7.6
0.931066	0.75	7.4
0.825	0.396447	7.4
0.825	1.103553	7.9
0.825	0.75	8.2
0.825	0.75	8.3

*These columns are automatically generated by the program.

Analyzing the Experimental Results

1. Choose SPECIAL... EXPERIMENTAL DESIGN... ANALYZE DESIGN... from the Menu bar to display the Analyze Design dialog box.

2. Enter ***Thickness*** in the Data text box, then click OK to display the Analysis Summary and Standardized Pareto Chart in the Analysis window. Maximize the Analysis Summary (see Figure 2-9).

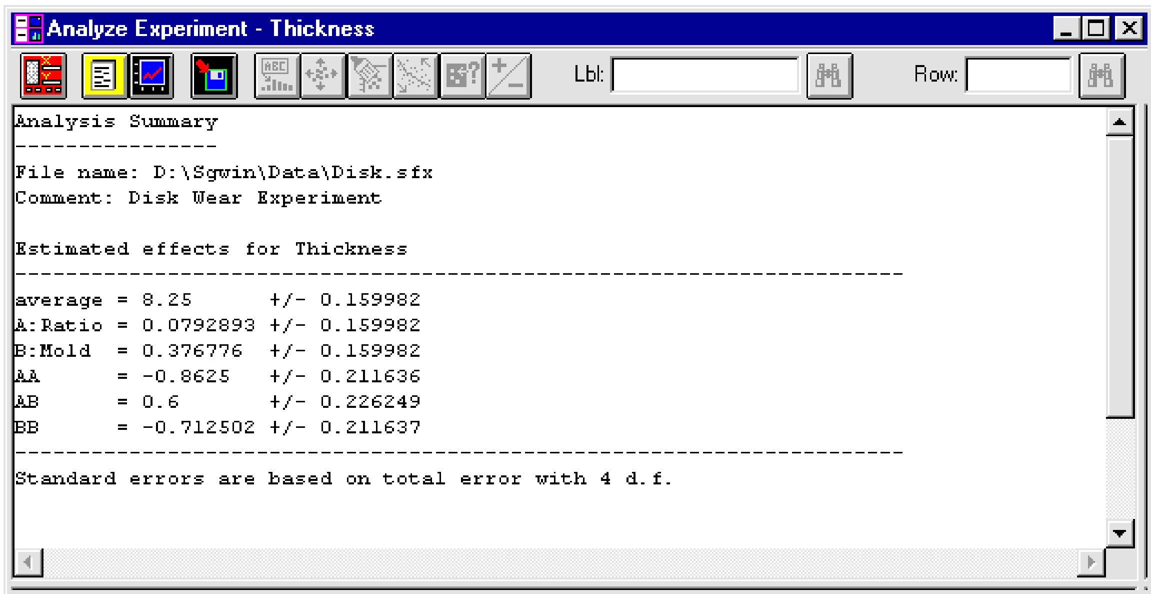


Figure 2-9. Analysis Summary

To determine the adequacy of the second-order model, you will look first at an ANOVA table.

3. Click the Tabular Options button to display the dialog box, then click the ANOVA Table option, and click OK to display the table in the second text pane. Maximize the table.

   The ANOVA Table shows that the quadratic terms (AA and BB) are significant (*p*-values $< .05$). To determine how well this model fits the data, run a lack-of-fit test.

4. Click the right mouse button on the text pane, then the left on Pane Options to display the Analysis of Variance Options dialog box.

5. Click the Include Lack-of-Fit check box (see Figure 2-10).

6. Click OK to redisplay the ANOVA Table (see Figure 2-11).

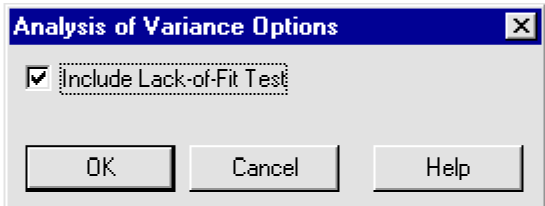
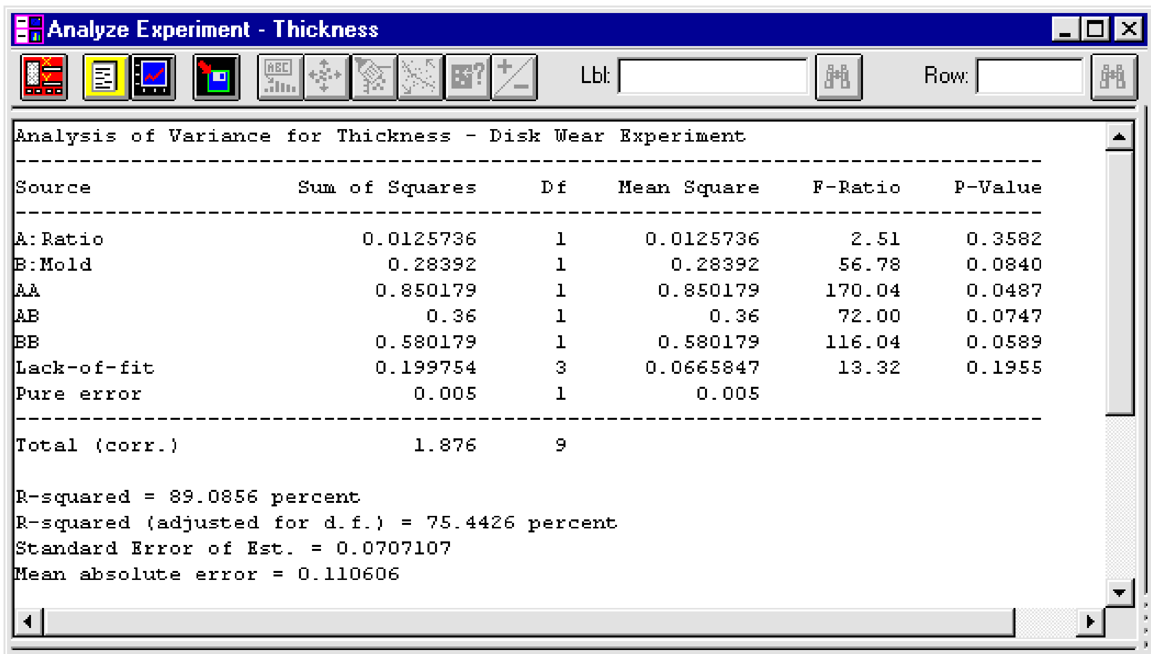


Figure 2-10. Completed Analysis of Variance Options Dialog Box



Analysis of Variance for Thickness - Disk Wear Experiment

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
A:Ratio	0.0125736	1	0.0125736	2.51	0.3582
B:Mold	0.28392	1	0.28392	56.78	0.0840
AA	0.850179	1	0.850179	170.04	0.0487
AB	0.36	1	0.36	72.00	0.0747
BB	0.580179	1	0.580179	116.04	0.0589
Lack-of-fit	0.199754	3	0.0665847	13.32	0.1955
Pure error	0.005	1	0.005		
Total (corr.)	1.876	9			

R-squared = 89.0856 percent
R-squared (adjusted for d.f.) = 75.4426 percent
Standard Error of Est. = 0.0707107
Mean absolute error = 0.110606

Figure 2-11. Redisplayed ANOVA Table

The program estimated the pure error from the replicated centerpoints, which are shown in the redisplayed table. Because the *p*-value for the lack-of-fit test is larger than .05, the second-order model seems to adequately fit the response.

7. Redisplay the Analysis of Variance Options dialog box and deselect the Lack-of-Fit Test check box to remove that option, then click OK to redisplay the original ANOVA Table.

8. Click the Graphical Options button to display the dialog box, then click the Normal Probability Plot of Effects check box, and OK to display the plot. Maximize the plot.

9. Click the right mouse button on the plot, then the left on Pane Options to display the Effects Normal Probability Plot Options dialog box.

10. Click the Label Effects check box (see Figure 2-12).

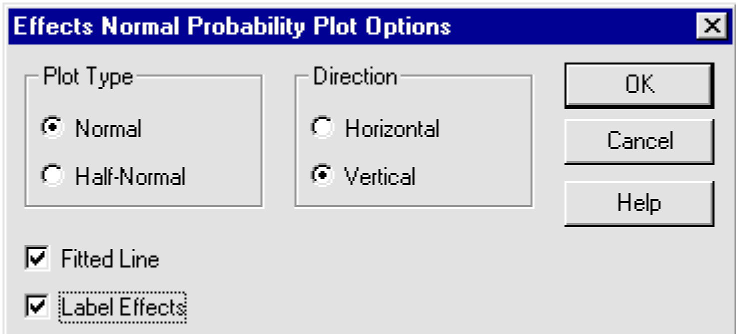


Figure 2-12. Completed Options Dialog Box

11. Click OK to redisplay the plot with the terms labeled (see Figure 2-13).

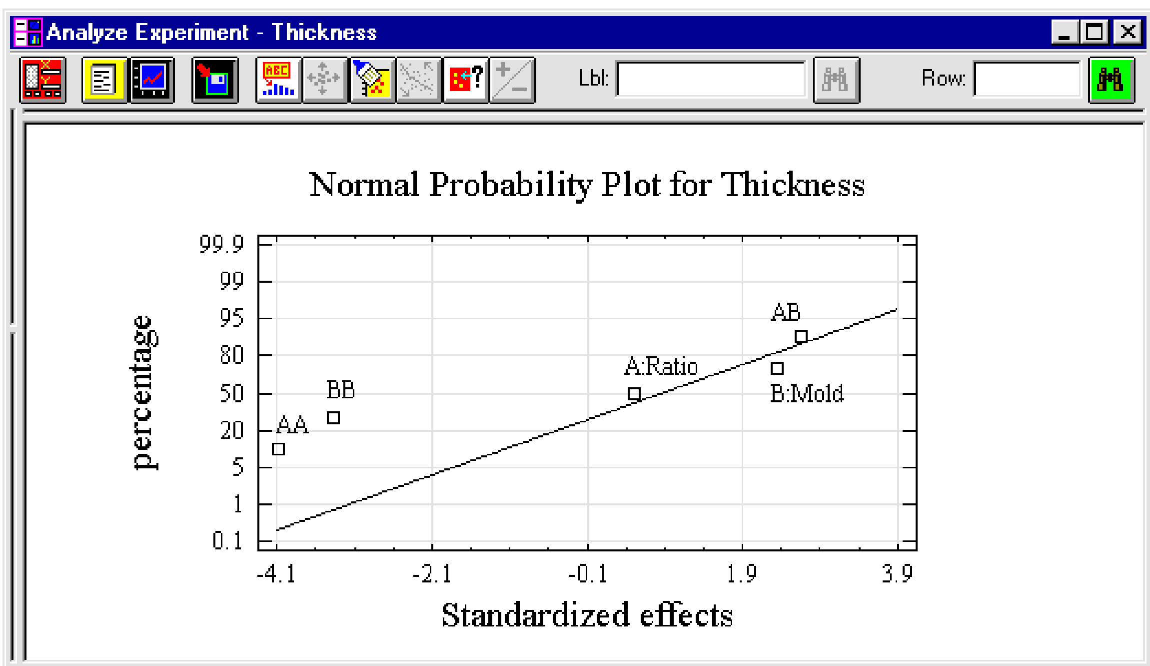


Figure 2-13. Normal Probability Plot with the Terms Labeled

The quadratic terms have fallen away from the fitted line, which indicates their significant effect on **Thickness**.

Further tests will probably help determine the adequacy of the second-order model; for example, look at the Standardized Pareto Chart.

12. Minimize the plot, then maximize the Pareto Chart in the first graphics pane (see Figure 2-14).

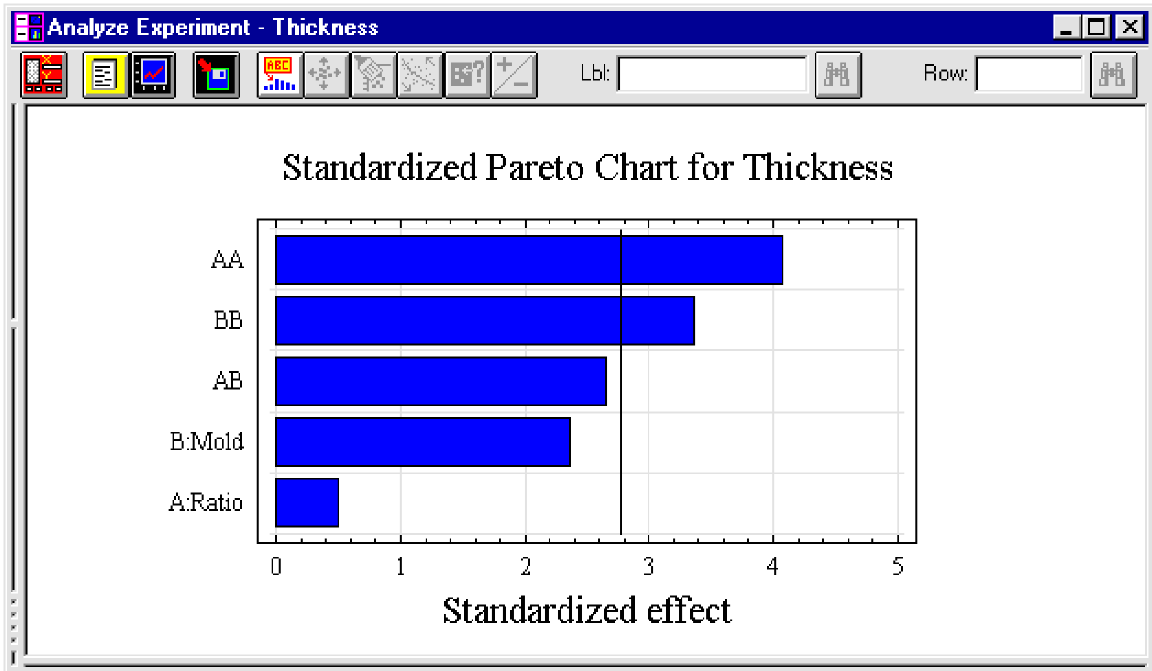


Figure 2-14. Standardized Pareto Chart

Again the quadratic terms are shown as significant effects; their associated bars cross the vertical line, which represents a 95 percent test of significance.

In the second phase of the experiment, the goal will be to locate the region of maximal thickness for the gasket by determining the coordinates for the factors that produce the largest values. A Surface Plot best illustrates this.

13. Click the Graphical Options button to display the dialog box, then click the first Response Plot option, and OK to display the plot. Maximize the plot in the third pane as shown in Figure 2-15.

This is a three-dimensional plot, which resembles a hill with peaks at levels of 0.9 for both **Ratio** and **Mold**. To view the plot from various angles, use the Smooth/Rotate button on the Analysis toolbar to rotate the plot

To see the results from another perspective, create a Contour Plot.

14. Click the Graphical Options button to display the dialog box, then click the second Response Plot option, and OK to display the plot in the fourth graphics pane. Maximize the plot.

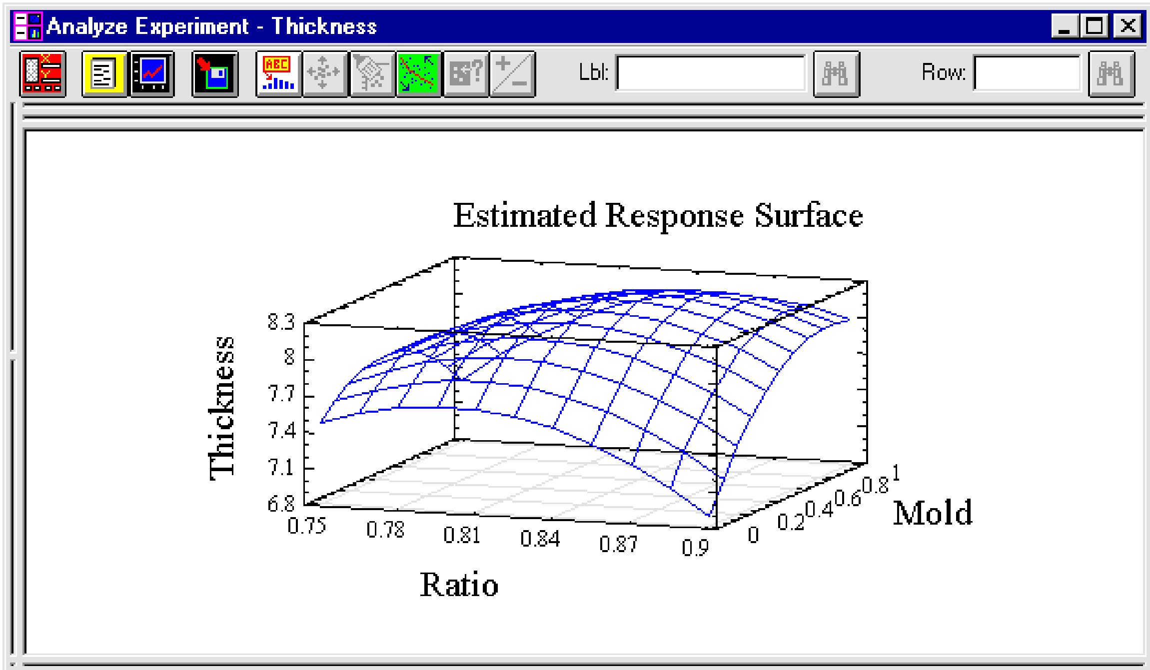


Figure 2-15. The Response Surface Plot

15. Click the right mouse button on the plot, then the left on Pane Options to display the Response Plot Options dialog box.

This dialog box lets you choose various ways in which to view the results.

16. Accept all the defaults except for the Contours options. Click the Painted Regions option (see Figure 2-16).

17. Click OK to redisplay the plot (see Figure 2-17).

The Contour Plot confirms that maximal disk thickness occurs when the position of the disk in the mold falls between .83 and .87, and when the ratios of filler to epoxy fall between .8 and .9.

The analysis has successfully shown that using a two-factor, central composite design adequately describes the process. In addition, it has determined the region of maximal response.

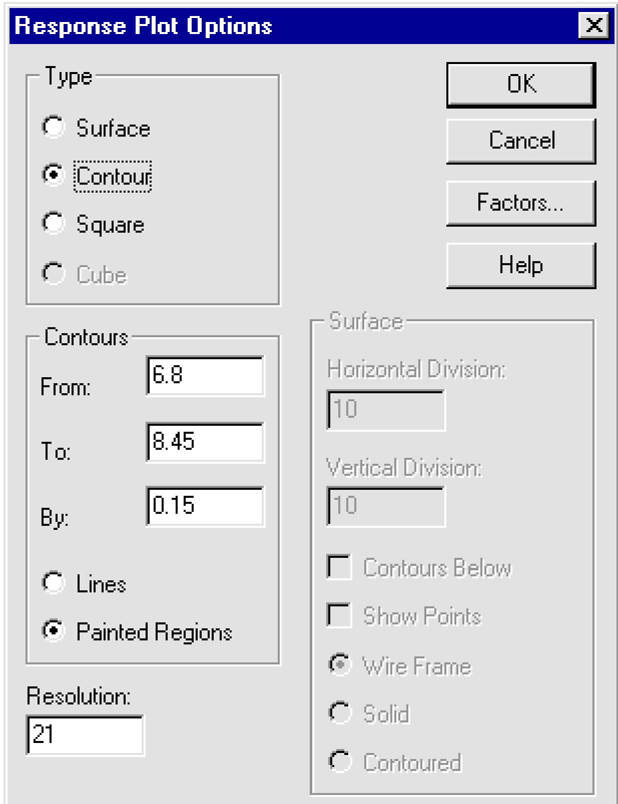


Figure 2-16. Completed Response Plot Options Dialog Box

References

Cornell, J. A. 1990. *How to Apply Response Surface Methodology*, vol 8 in *Basic References in Quality Control: Statistical Techniques*, edited by S. S. Shapiro and E. Mykytka. Milwaukee: American Society for Quality Control.

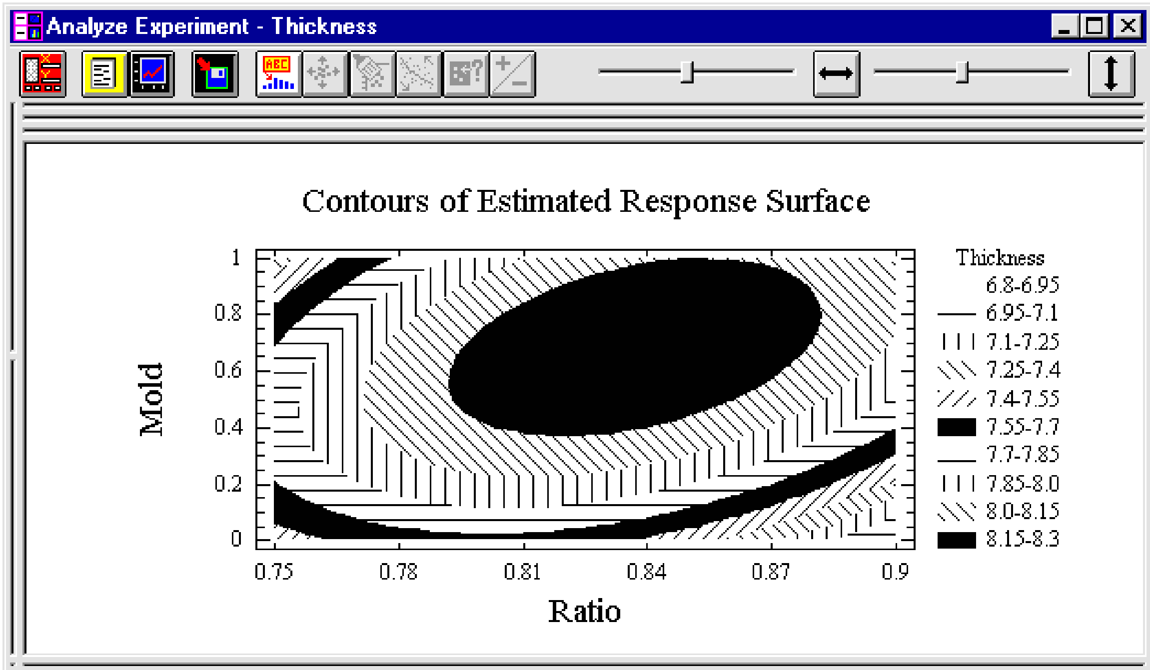


Figure 2-17. Contour Plot

TUTORIAL 3

Creating and Analyzing a Mixture Design

This tutorial is based on Kurotori's (1966) rocket propellant experiment and Snee's (1971) analysis of it. The rocket propellant is made up of a combination of a binder, an oxidizer, and fuel. The combination of these ingredients creates a propellant with properties such as elasticity.

You want to mix the components to achieve an elasticity of 3000. You also want to determine the formula that best combines these components and find a model that can predict elasticity. You will set up a simplex-centroid design with check blends. The design has 10 experimental runs.

To begin, open STATGRAPHICS *Plus*.

Creating the Design

1. Choose SPECIAL... EXPERIMENTAL DESIGN... CREATE DESIGN... from the Menu bar to display the Create Design Options dialog box.

2. Click the Mixture button under Design Class options.

3. Accept the defaults in the Number of Response Variables and Number of Components text boxes.

4. Type ***Rocket Propellant Study*** in the Comment text box.

This name will appear on reports and graphs when you analyze the experiment. Your dialog box should now look like the one shown in Figure 3-1.

5. Click OK to display the Component Definition Options dialog box.

In this example, you need all three components to make the rocket propellant, so you will include all three in the blend.

6. Complete the dialog box for Component A: For Component A, type ***Binder*** in the Name text box, type ***0.2*** in the Low text box, type ***0.4*** in the High text box, accept the default in the Mixture Total text box, and type ***%*** in the Units text box (see Figure 3-2).

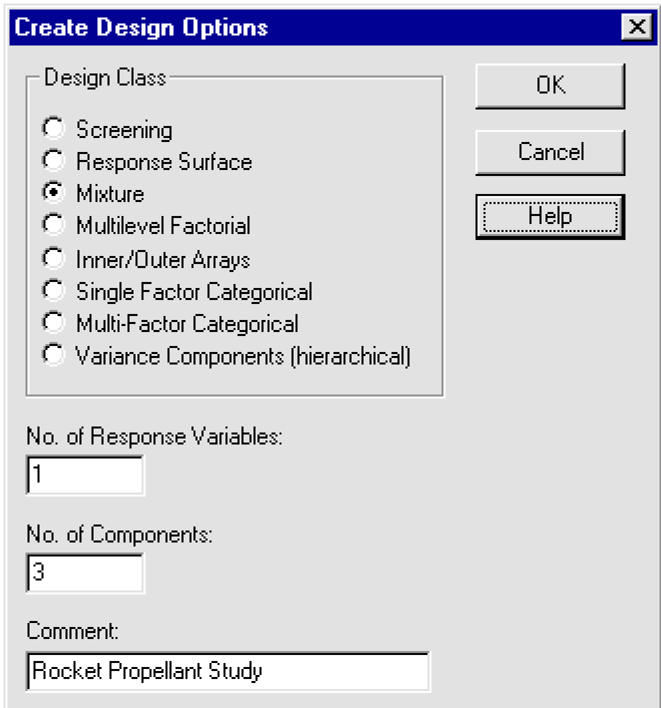


Figure 3-1. Completed Create Design Options Dialog Box (Mixture Design)

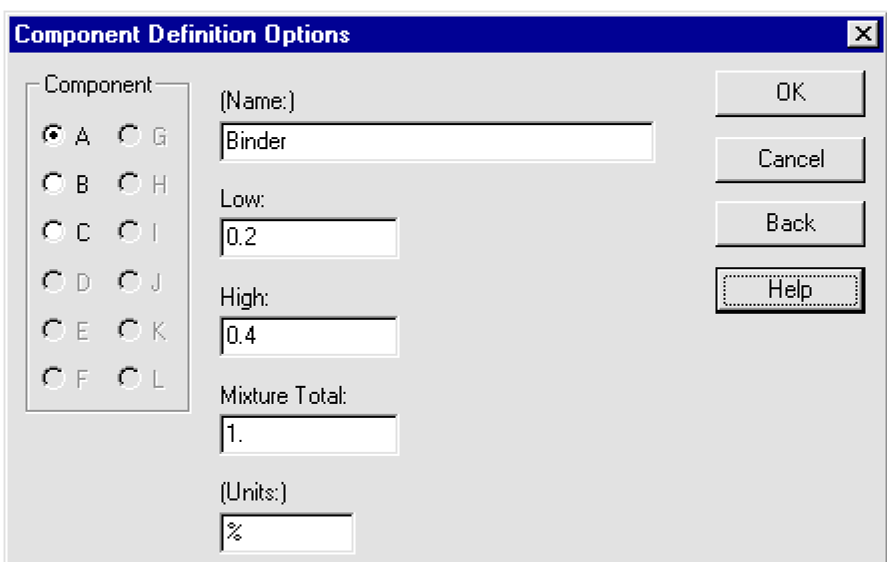


Figure 3-2. Completed Dialog Box for Component A

7. Complete the dialog box for Component B: Click the Component B option, then type ***Oxidizer*** in the Name text box, type ***0.4*** in the Low text box, type

0.6 in the High text box, accept the default in the Mixture Total text box, and delete ***5*** from the Units text box so it is blank (see Figure 3-3).

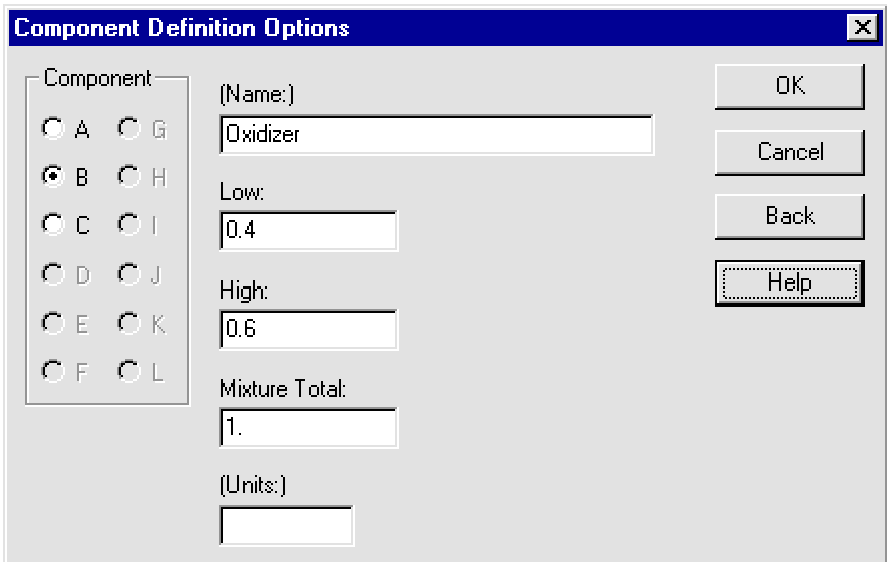


Figure 3-3. Completed Dialog Box for Component B

8. Complete the dialog box for Component C: Click the Component C option, then type ***Fuel*** in the Name text box, type ***0.2*** in the Low text box, type ***0.4*** in the High text box, accept the default in the Mixture Total text box, and leave the Units text box blank (see Figure 3-4).

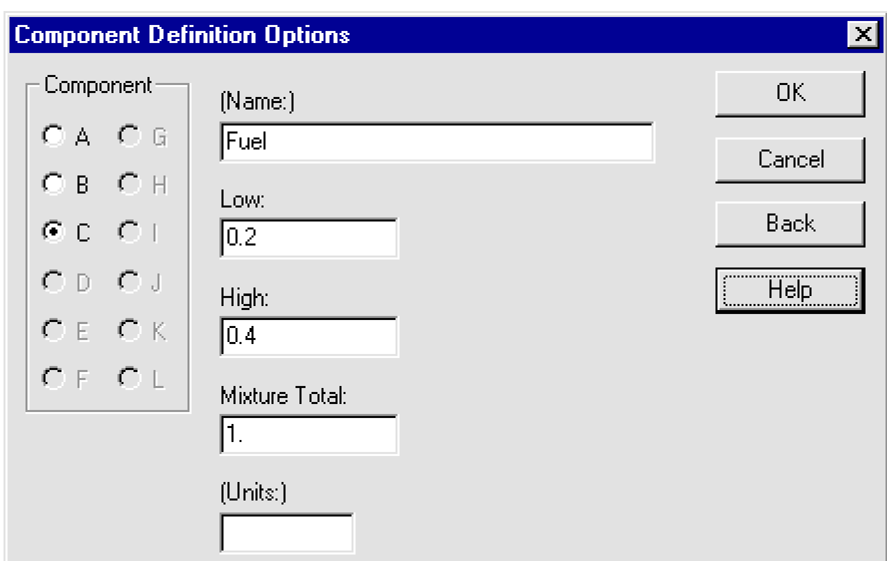


Figure 3-4. Completed Dialog Box for Component C

9. Click OK to display the Response Definition Options dialog box.

10. Type ***Elasticity*** in the Name text box and leave the Units text box blank (see Figure 3-5).

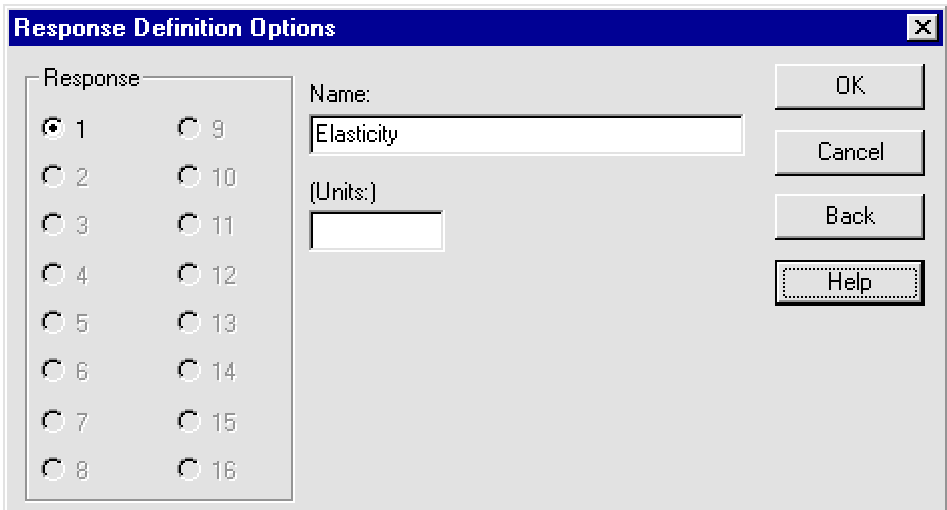


Figure 3-5. Completed Response Definition Options Dialog Box

11. Click OK to display the Mixture Design Selection dialog box.

12. Use the down arrow to display the available designs, then choose Simplex-Centroid (see Figure 3-6).

13. Click OK to display the Mixture Design Options dialog box.

    Notice that the name of the base design appears on the first line of the dialog box. Beneath that, the number of runs is displayed.

14. Click the Special Cubic option under Model Type, accept the default in the Number of Replicate Points text box, click the Augment Design check box, and deselect Randomize (see Figure 3-7).

15. Click OK to display the Design Summary in the Mixture Design Attributes window. Maximize the summary (see Figure 3-8).

    The Design Summary includes the name of the design and its class, and the comment you entered about the experiment. If you have not yet saved the design the file name will appear as <Untitled>. The rest of the information summarizes the components, responses, and runs.

    After reviewing the details of the design, save it.

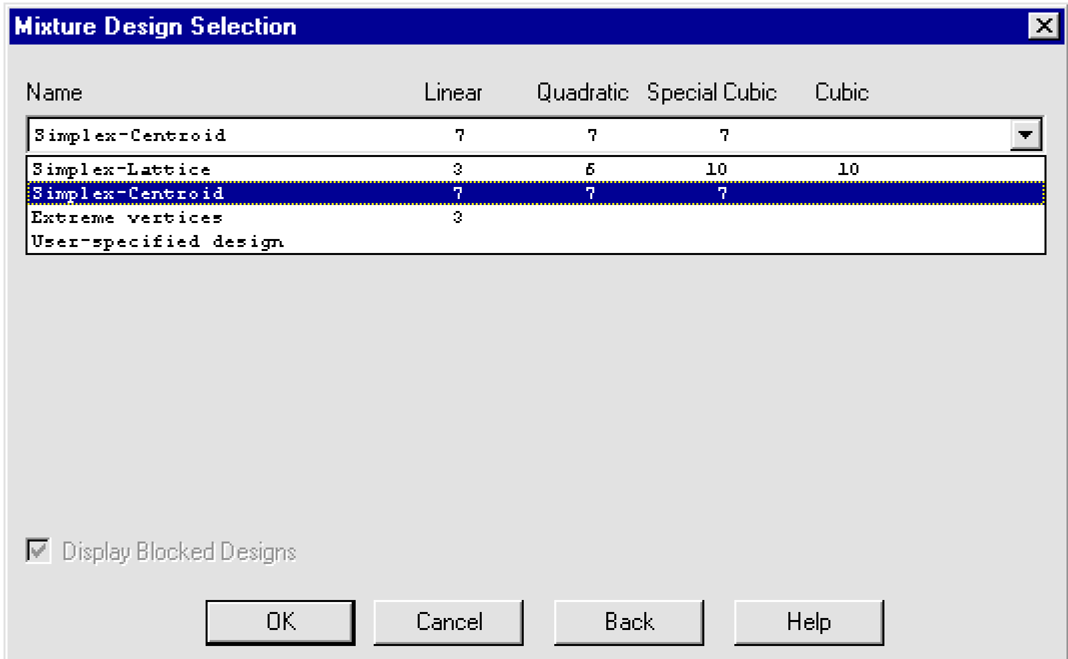


Figure 3-6. Mixture Design Selection Dialog Box with Chosen Design

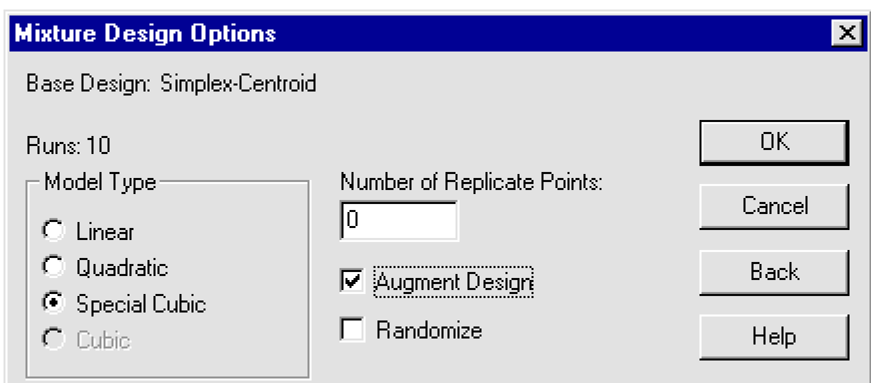


Figure 3-7. Completed Mixture Design Options Dialog Box

Naming and Saving the Design

1. Choose FILE... SAVE AS... SAVE DESIGN FILE AS... from the Menu bar to display the Save Design As dialog box.

2. Type ***Rocket.sfx*** in the Name text box and click the Save button to name and save the file and redisplay the Design Summary.

Now you are ready to collect and set up the data for the experiment. The design and response values are shown in Table 3-1.

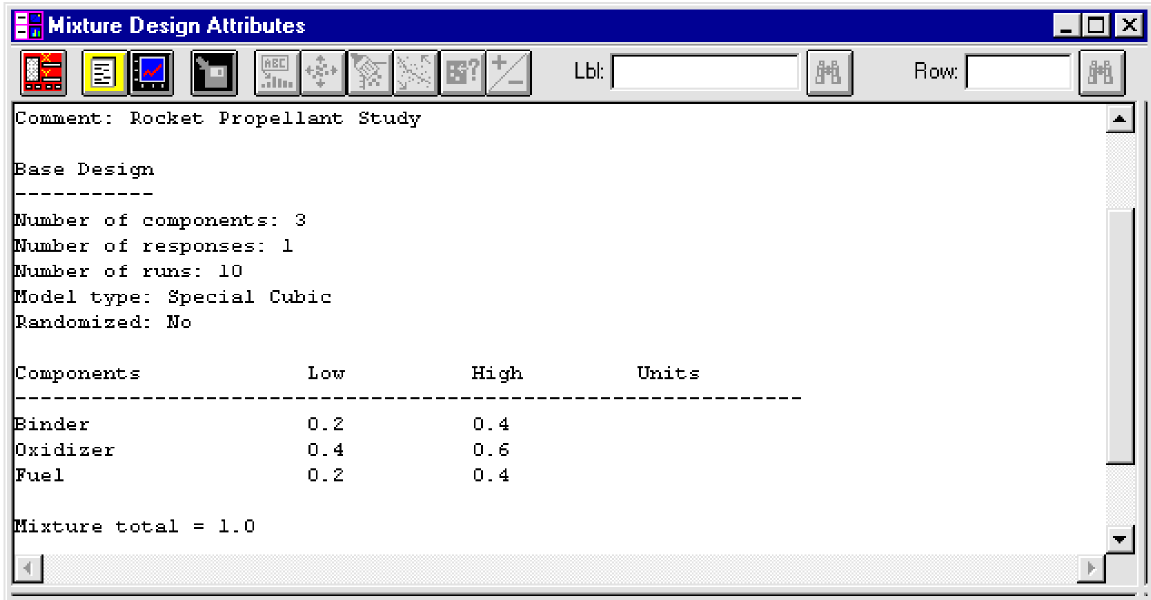


Figure 3-8. Design Summary

3. Enter the values for **Elasticity** from Table 3-1.

4. Resave the design.

Table 3-1. Design and Response Values for the Rocket.sfx File

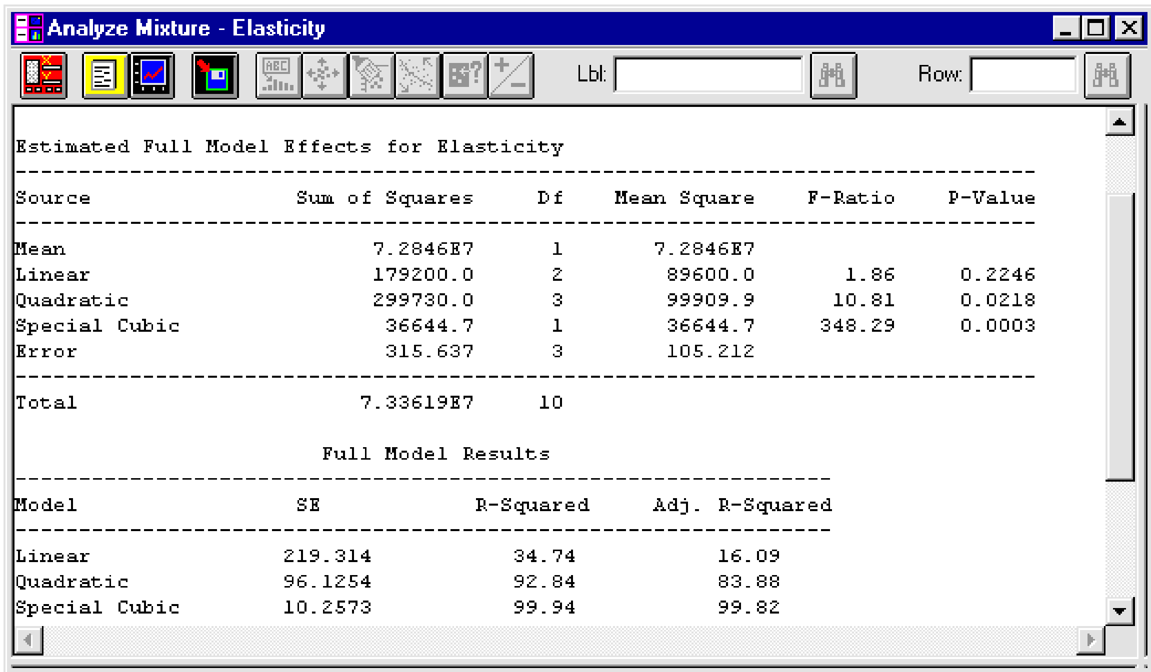
Binder	*Oxidizer*	*Fuel*	*Elasticity*
0.4	0.4	0.2	2350.0
0.2	0.6	0.2	2450.0
0.2	0.4	0.4	2650.0
0.3	0.5	0.2	2400.0
0.3	0.4	0.3	2750.0
0.2	0.5	0.3	2950.0
0.266667	0.466667	0.266667	3000.0
0.333333	0.433333	0.233333	2690.0
0.233333	0.533333	0.233333	2770.0
0.233333	0.433333	0.333333	2980.0

Now you are ready to analyze the design.

Analyzing the Experimental Results

1. Choose SPECIAL... EXPERIMENTAL DESIGN... ANALYZE DESIGN... from the Menu bar to display the Analyze Design dialog box.

2. Enter ***Elasticity*** into the Data text box and click OK to display the Analysis Summary and the Estimated Response Surface Plot in the Analysis window. Maximize the Analysis Summary (see Figure 3-9).



Estimated Full Model Effects for Elasticity

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Mean	7.2846E7	1	7.2846E7		
Linear	179200.0	2	89600.0	1.86	0.2246
Quadratic	299730.0	3	99909.9	10.81	0.0218
Special Cubic	36644.7	1	36644.7	348.29	0.0003
Error	315.637	3	105.212		
Total	7.33619E7	10			

Full Model Results

Model	SE	R-Squared	Adj. R-Squared
Linear	219.314	34.74	16.09
Quadratic	96.1254	92.84	83.88
Special Cubic	10.2573	99.94	99.82

Figure 3-9. Analysis Summary

To determine if the Special Cubic model is the most appropriate model to fit to the ***Elasticity*** data, look at the estimated full model results in the Analysis Summary pane.

The results show the conditional sum of squares, degrees of freedom, mean squares, F-ratios, and *p*-values for each of the polynomial model types. The results indicate that the Quadratic, Special Cubic, and Linear models are statistically significant (*p*-value less than .05). The results also show the SE (square root of MSE), R-squared, and adjusted R-squared values. The SE is smaller and the adjusted R-squared value is larger for the Special Cubic model than it is for the Linear and Quadratic models. These results indicate that the Special Cubic model is better than the Linear and Quadratic models for estimating the elasticity.

The Special Cubic model is the best overall model with respect to SE and Adjusted R-squared values. You will continue the analysis using the Special Cubic model.

3. Click the Tabular Options button to display the dialog box, then click the Model Results check box and OK to display the model-fitting results in the second text pane. Maximize the pane (see Figure 3-10).

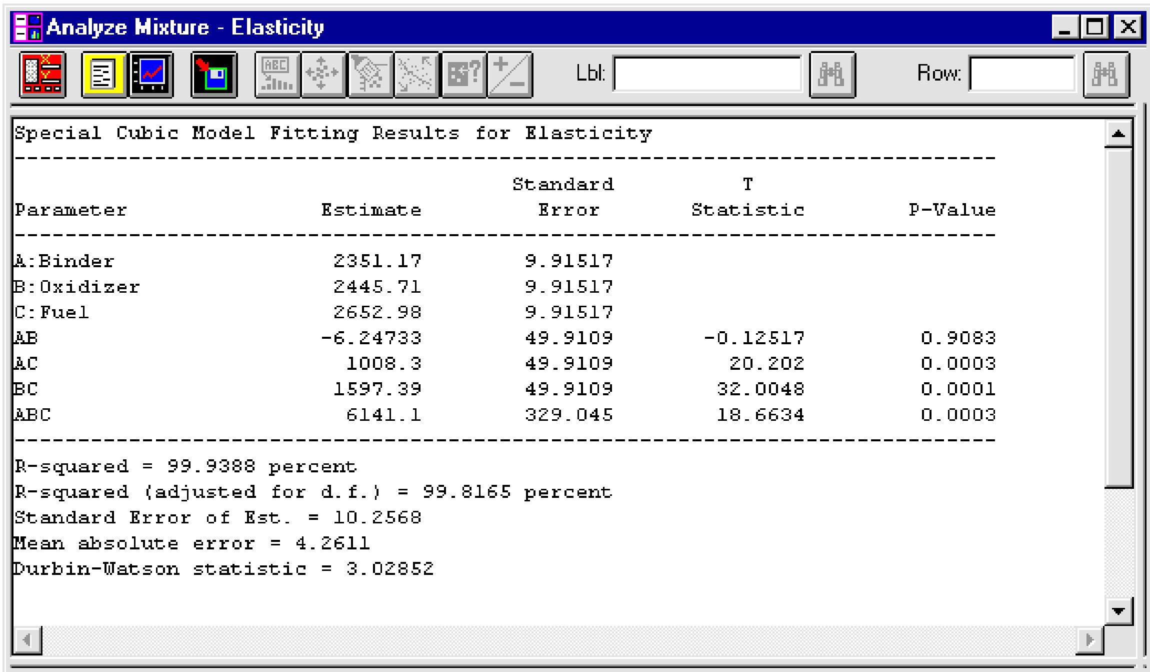


Figure 3-10. Model-Fitting Results for the Special Cubic Model

The *p*-value for the AB term is greater than .05, while the values for other terms are less than .05. Therefore, all terms in the model are significant except the AB term. To test for overall model significance, produce an ANOVA Table.

4. Click the Tabular Options button to display the dialog box, click the ANOVA Table check box, then OK to display the table. Maximize the pane (see Figure 3-11).

The ANOVA Table shows that the *p*-value is less than .05, which indicates that the overall model is highly significant. Now, look at two Response Plots: a Surface Plot and a Contour Plot.

5. Click the Graphical Options button to display the dialog box, then click the two Response Plot check boxes. Maximize the top Response Plot (see Figure 3-12).

Analyze Mixture - Elasticity

ANOVA for Elasticity - Rocket Propellant Study

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Special Cubic Model	515574.0	6	85929.1	816.80	0.0001
Total error	315.607	3	105.202		
Total (corr.)	515890.0	9			

R-squared = 99.9388 percent
R-squared (adjusted for d.f.) = 99.8165 percent

Figure 3-11. The ANOVA Table

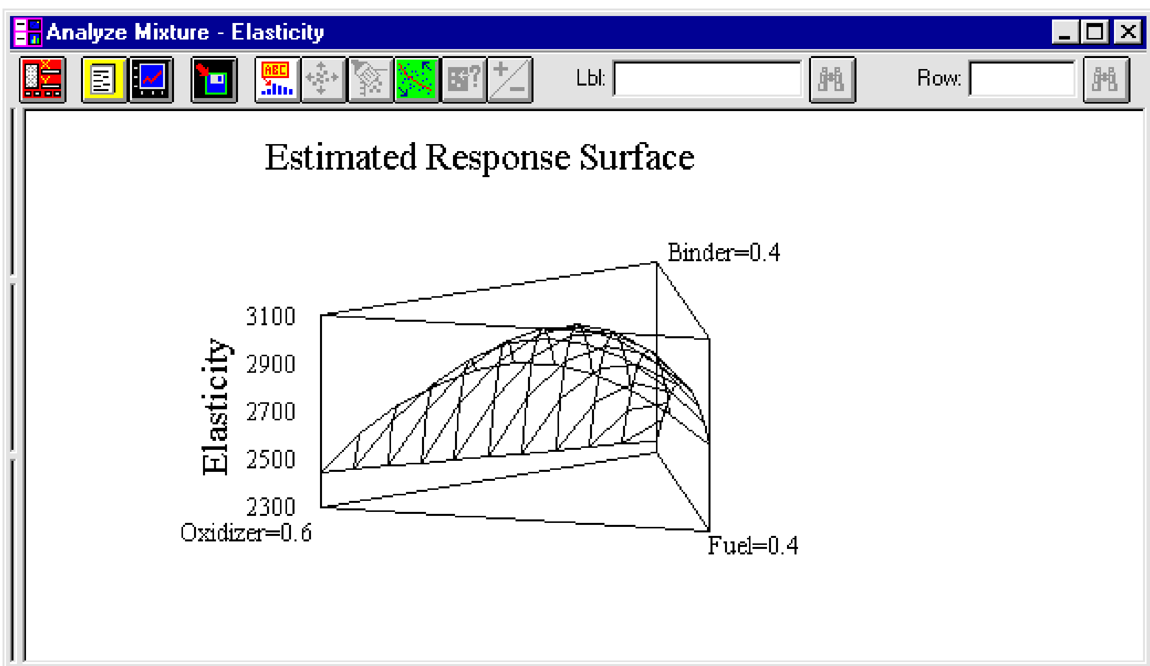


Figure 3-12. The Response Surface Plot

The Response Surface Plot displays the blends for the components as the height and shape of the response inside the experimental region. The plot is helpful in visualizing the location of the minimum and maximum response values.

Now look at the Contour Plot to find the height of the surface at various blends of the three components. The Contour Plot is a two-dimensional representation of the surface.

6. Minimize the Surface Plot and maximize the Contour Plot (see Figure 3-13).

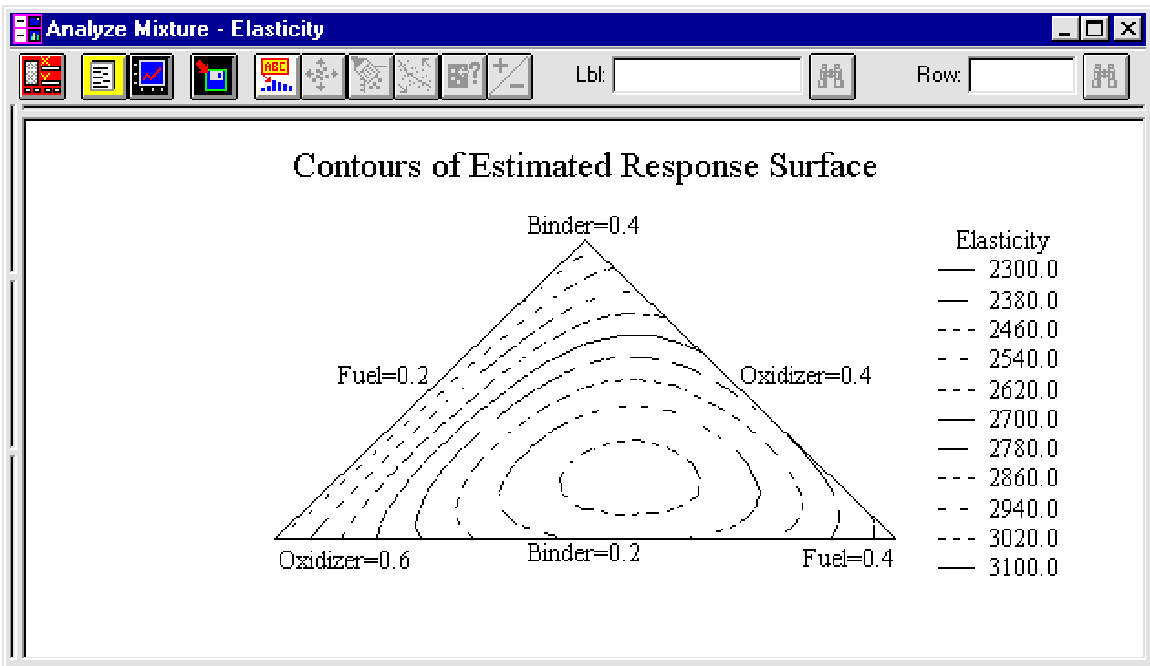
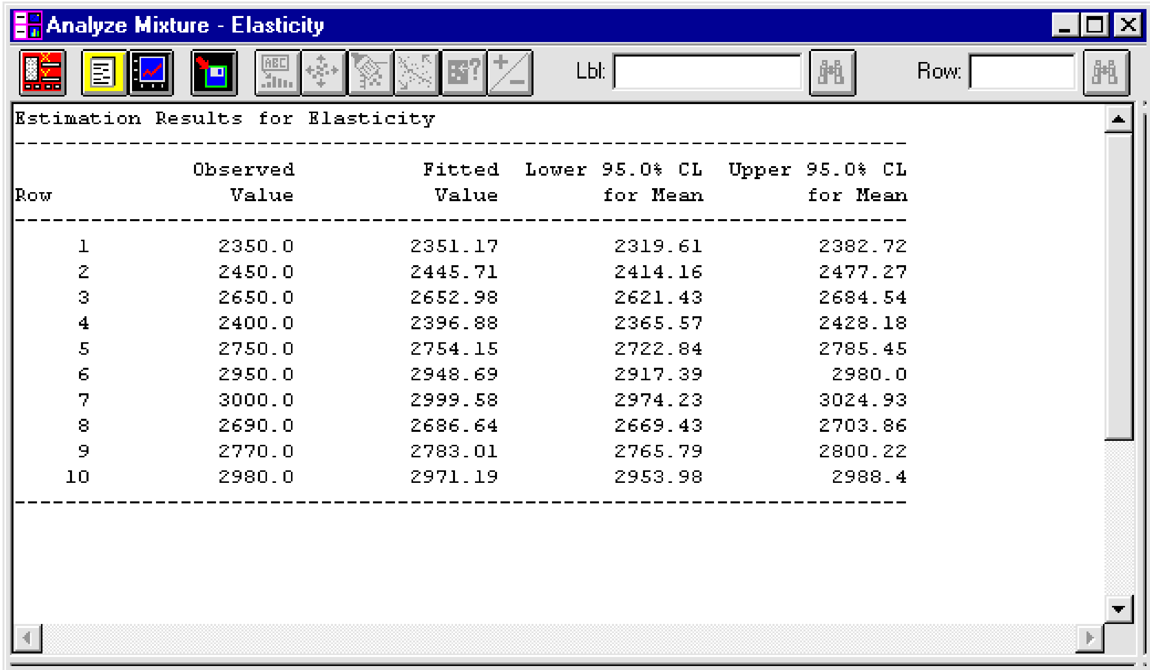


Figure 3-13. The Contour Plot

The Contour Plot shows the estimated elasticity values in the Mixture Design experimental region. From the plot you can see that the elasticity of 3000 is near the region where Binder = 0.25, Oxidizer = 0.45, and Fuel = 0.3. Therefore, these are the proportions of the components required to make a rocket propellant with an elasticity of 3000. The original data had an elasticity of 3000, where Binder = 0.266667, Oxidizer = 0.466667, and Fuel = 0.266667.

To check these results further, you can estimate the predicted elasticity with the Predictions option.

7. Click the Tabular Options button to display the dialog box, then click the Predictions check box, and OK to display the results. Maximize the table (see Figure 3-14).



Estimation Results for Elasticity

Row	Observed Value	Fitted Value	Lower 95.0% CL for Mean	Upper 95.0% CL for Mean
1	2350.0	2351.17	2319.61	2382.72
2	2450.0	2445.71	2414.16	2477.27
3	2650.0	2652.98	2621.43	2684.54
4	2400.0	2396.88	2365.57	2428.18
5	2750.0	2754.15	2722.84	2785.45
6	2950.0	2948.69	2917.39	2980.0
7	3000.0	2999.58	2974.23	3024.93
8	2690.0	2686.64	2669.43	2703.86
9	2770.0	2783.01	2765.79	2800.22
10	2980.0	2971.19	2953.98	2988.4

Figure 3-14. Predictions Test Results

The table displays the predicted values, 95 percent confidence intervals, 95 percent prediction intervals, and the observed values.

References

Kurtori, I. S. 1966. "Experiments with Mixtures of Components Having Lower Bounds," *Industrial Quality Control*, **22**:592-596.

Tutorial 4

AUGMENTING A DESIGN

The example in this tutorial is based on Box, Hunter, and Hunter's (1978) Resolution III bicycle experiment. In it, a two-level factorial screening design with one response variable and seven experimental factors has been created for you. The design is a fully saturated, sixteenth fraction design (2^{7-4}). Because it is a Resolution III design, the main effects are confounded with the second-order interactions.

The design requires only eight runs. After you identify the two or three most significant factors based on these runs, you will augment the design by adding a second fraction that de-aliases the confounding between one of the main effects and its associated second-order interactions.

In the exercise, a cyclist is training to compete in an uphill bicycle race and trying to decrease the time it takes to ride uphill. During practice, it was noted that several conditions vary. Determining which of these factors is most significant will help to plan a better training program.

Before you begin, open STATGRAPHICS *Plus* and the **Bicycle.sfx** file.

Analyzing the Experimental Results

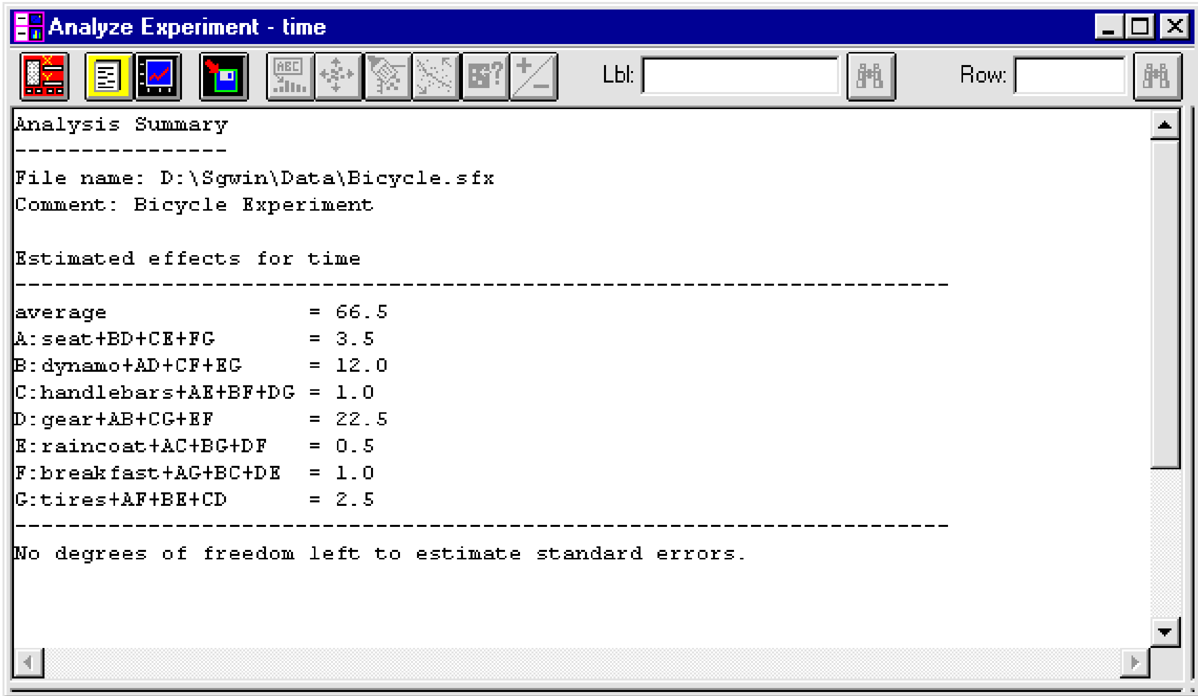
1. Choose SPECIAL... EXPERIMENTAL DESIGN... ANALYZE DESIGN... from the Menu bar to display the Analyze Design dialog box.

2. Enter **Time** into the Data text box and click OK to display the Analysis Summary and Pareto Chart in the Analysis window. Maximize the Analysis Summary (see Figure 4-1).

Notice that several two-factor interactions are confounded with the main effects.

Your next step is to determine which factors are more significant than the others. However, you cannot perform the test because there are no degrees of freedom available to estimate the error.

The **Dynamo** and **Gear** effects seem to be significant (that is, their estimated effects are larger than the effects for the other factors). Because the design is Resolution III, the Dynamo and Gear effects are confounded with second-order interactions. At this point, you cannot be certain if the observed effects are due to the main effects or to the second-order interactions. You can graphically display the size of the effects.




```
Analysis Summary
----------------
File name: D:\Sgwin\Data\Bicycle.sfx
Comment: Bicycle Experiment

Estimated effects for time
----------------------------------------------------------------------
average                   = 66.5
A:seat+BD+CE+FG           = 3.5
B:dynamo+AD+CF+EG         = 12.0
C:handlebars+AE+BF+DG = 1.0
D:gear+AB+CG+EF           = 22.5
E:raincoat+AC+BG+DF       = 0.5
F:breakfast+AG+BC+DE      = 1.0
G:tires+AF+BE+CD          = 2.5
----------------------------------------------------------------------
No degrees of freedom left to estimate standard errors.
```

Figure 4-1. Analysis Summary

3. Maximize the Pareto Chart (see Figure 4-2).

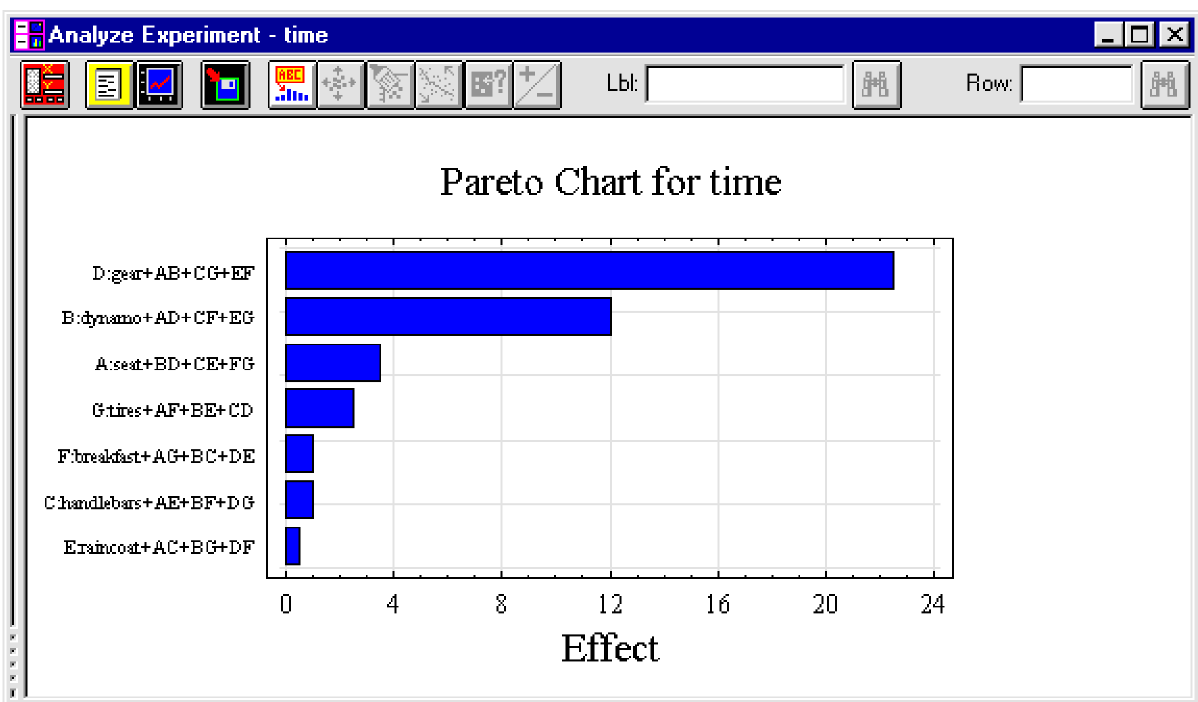


Figure 4-2. The Pareto Chart

The chart shows that Factor D (gear), which has the longest bar, has the largest effect on **Time**, and that Factor B (**Dynamo**), which has the second largest bar, has the second largest effect.

Now plot the values for the **Time** response variable for different levels of **Gear** and **Dynamo**. Before you do this, you must remove the confounding effects from the model.

4. Click the right mouse button on the plot, then the left on Analysis Options to display the Estimate Effects Options dialog box.

5. Type ***1*** in the Maximum Order Effect text box, and accept the defaults for the remaining options on the dialog box (see Figure 4-3).

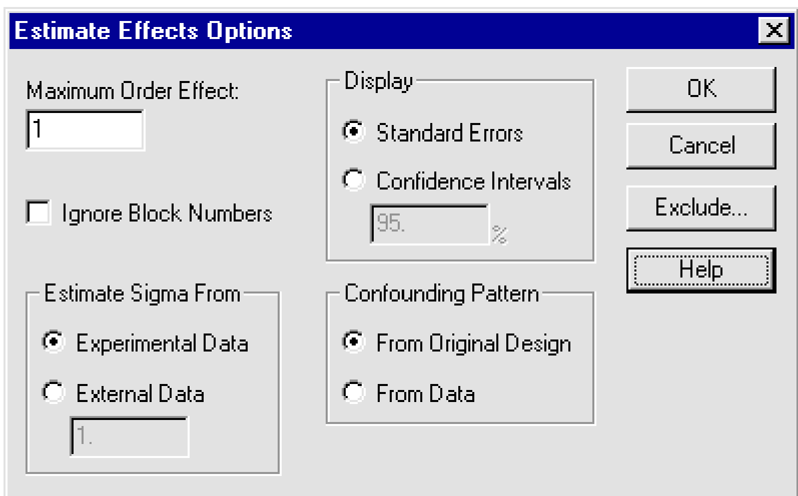


Figure 4-3. The Completed Estimate Effects Options Dialog Box

6. Click OK to remove the two-factor interactions and redisplay the Analysis Summary and Pareto Chart in the Analysis window (see Figure 4-4).

Now that you have removed the confounding effects, you can plot the values.

7. Click the Graphical Options button to display the dialog box, then click the check box for the first Response Plots option, then click OK to display the Analysis Summary and the two plots in the Analysis window (see Figure 4-5).

8. Click the right mouse button on the Response Plot then the left on Pane Options to display the Response Plot Options dialog box.

9. Click the Square Plot option under the Type portion of the dialog box, and accept the defaults for the remainder of the dialog boxes (see Figure 4-6)

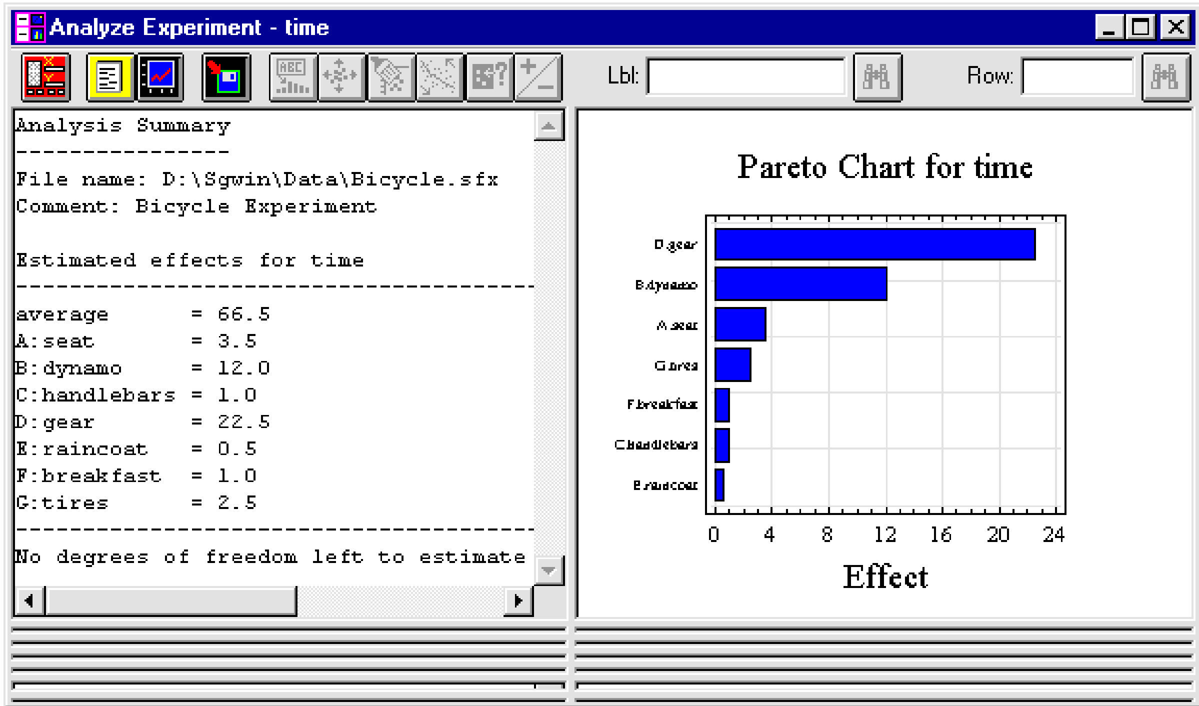


Figure 4-4. The Analysis Summary and Pareto Chart with the Two-Factor Interactions Removed

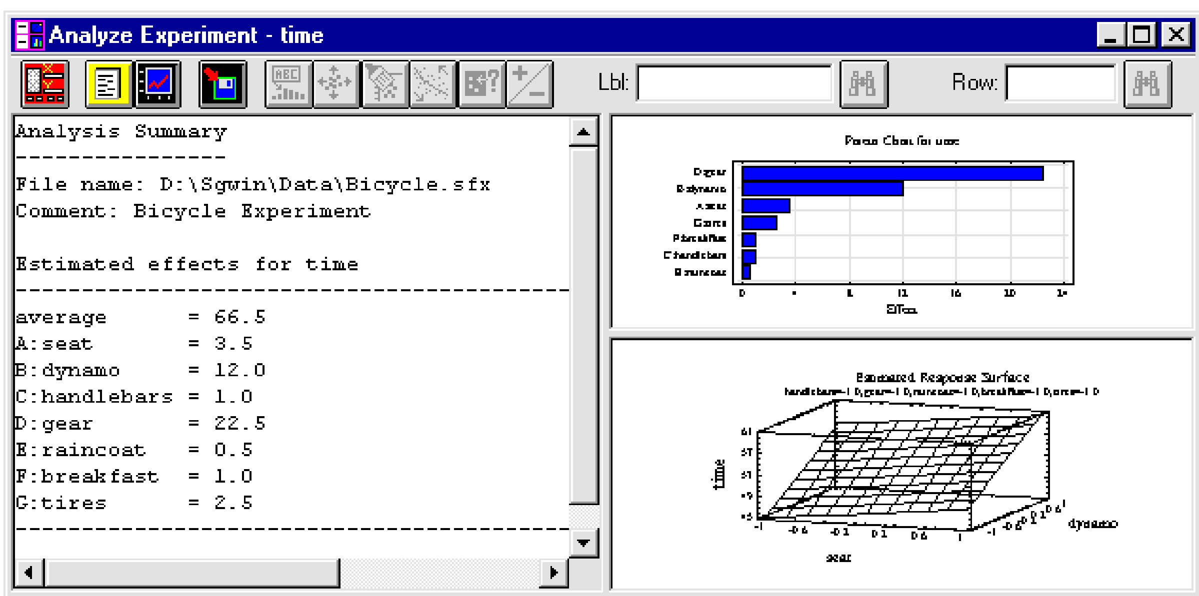


Figure 4-5. Analysis Summary and Two Plots Displayed in the Analysis Window

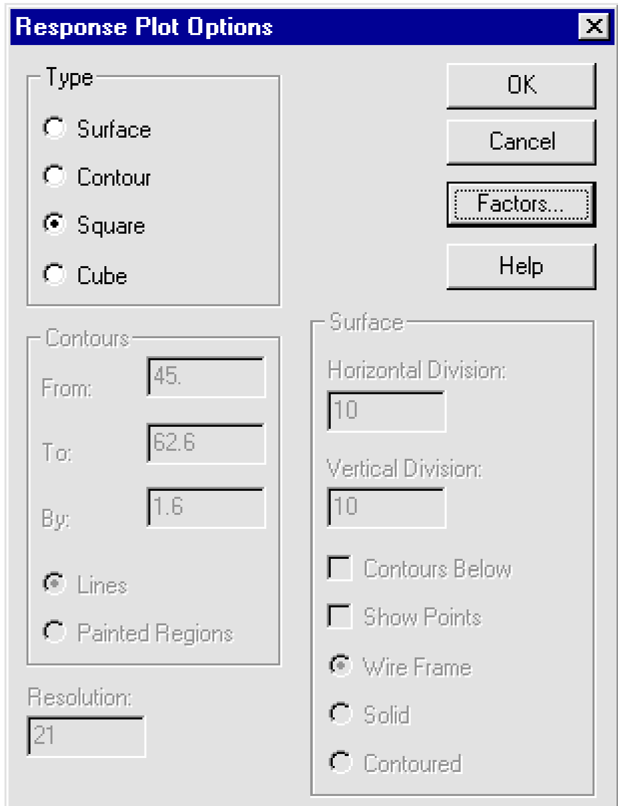


Figure 4-6. Completed Response Plot Options Dialog Box

10. Click the Factors command on the Response Plot Options dialog box to display the Response Plot Factors dialog box.

11. Click **Seat** to deselect that variable, accept **Dynamo** as the default, and click **Gear** (see Figure 4-7).

12. Click OK, first on the Response Plot Factors dialog box then on the Response Plot Options dialog box to display the Square Plot in the Analysis window. Maximize the plot (see Figure 4-8).

The Square Plot displays the values for the predicted response variable, **Time**, at the different combinations of the factors **Dynamo** and **Gear**. Use the plot to identify the combination that produces the best value for the response variable.

The plot illustrates that it takes considerably longer to ride up hill when the dynamo is on than when it off. It also shows that it takes longer to ride up hill in medium gear than in low gear.

To investigate further, you will augment the experiment.

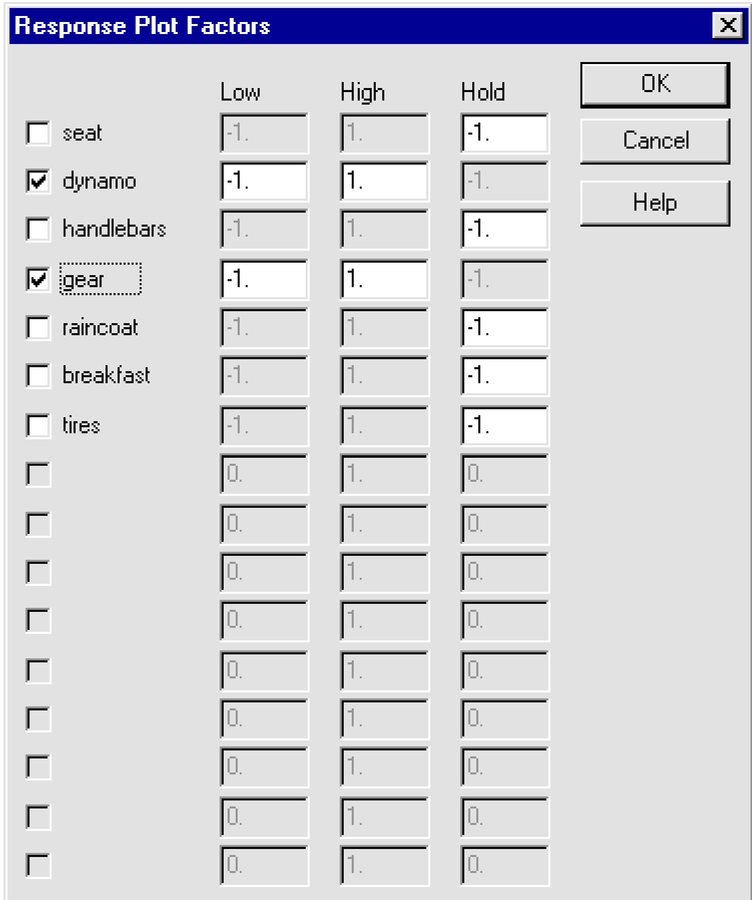


Figure 4-7. Completed Response Plot Factors Dialog Box

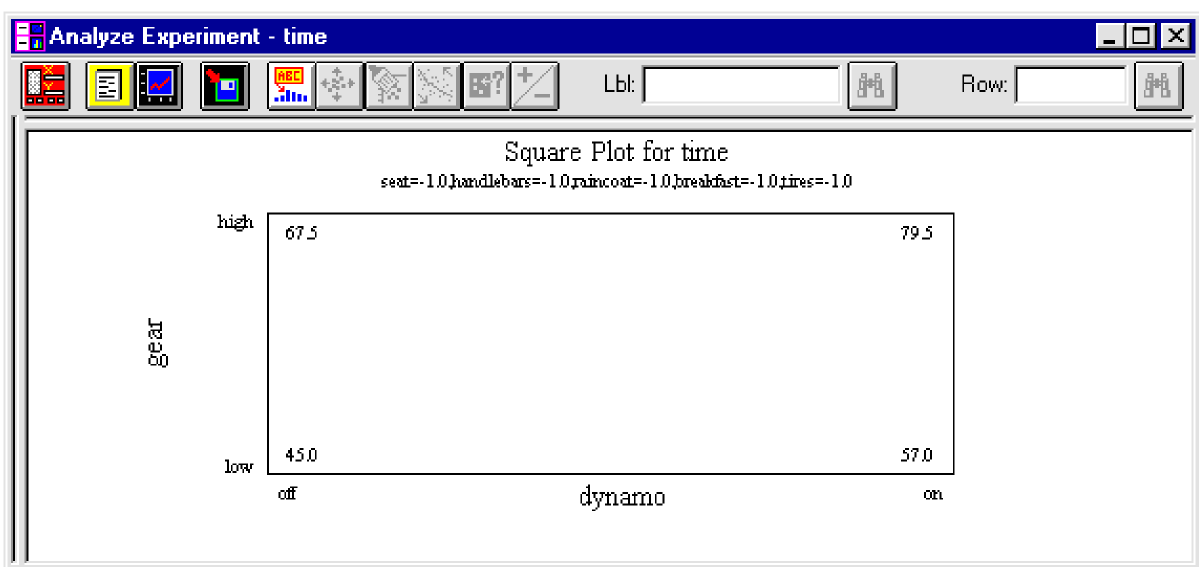


Figure 4-8. The Square Plot

Augmenting the Experimental Results

1. Choose SPECIAL... EXPERIMENTAL DESIGN... AUGMENT DESIGN... from the Menu bar to display the Augment Design dialog box.

2. Choose the Clear a Factor option (see Figure 4-9).

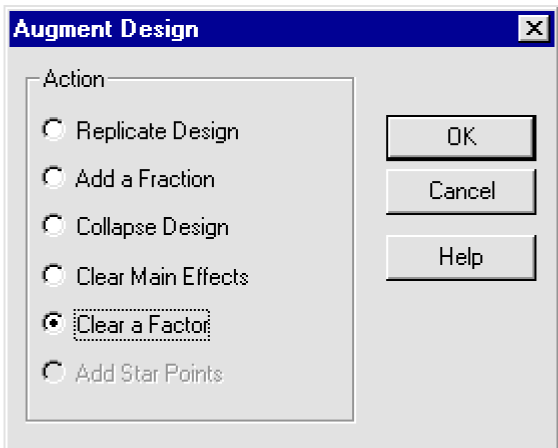


Figure 4-9. Completed Augment Design Dialog Box

3. Click OK to display the Clear a Factor dialog box.

4. Click the **Gear** option (see Figure 4-10).

5. Click OK to display the Design Summary (see Figure 4-11).

The Design Summary shows that **Gear** is cleared from confounding with two-factor interactions and that all the other two-factor interactions are also cleared.

Now you will save the augmented design using a new name.

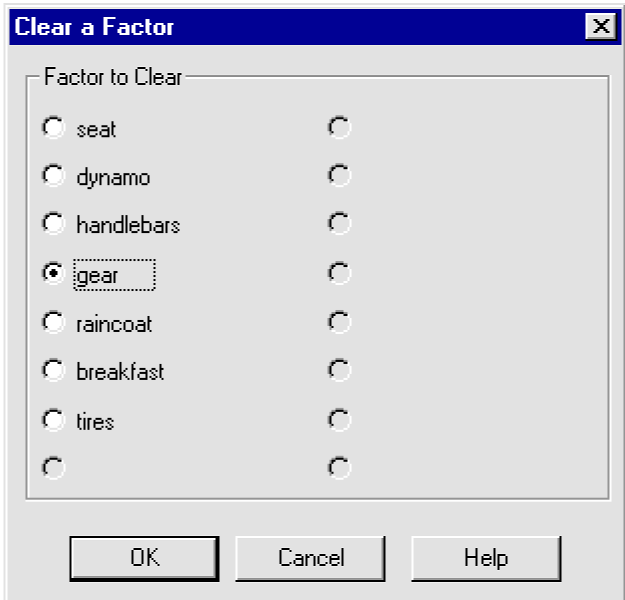


Figure 4-10. Completed Clear a Factor Dialog Box

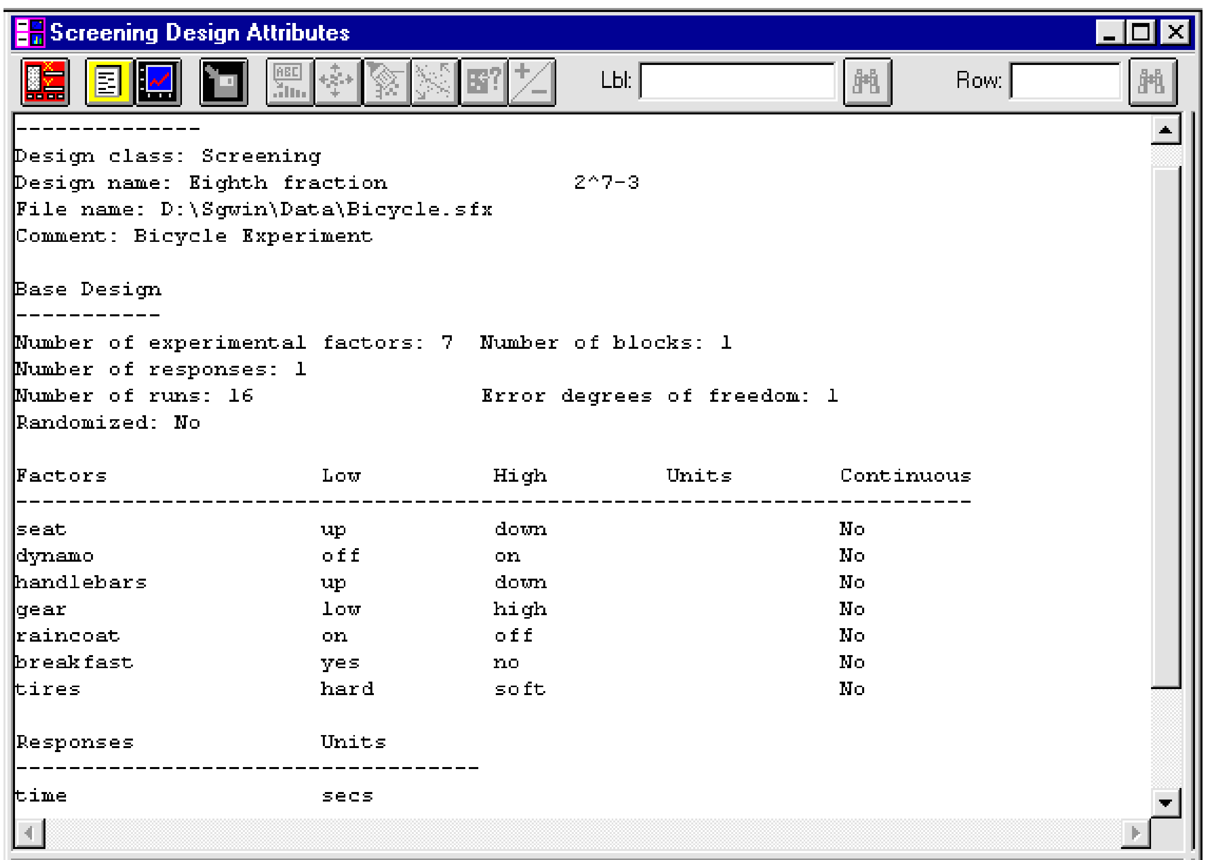


Figure 4-11. The Design Summary Following Augmentation

Saving the Augmented Design

1. Choose FILE... SAVE AS... SAVE DESIGN FILE AS... from the Menu bar to display the Save Design File As dialog box.

2. Type ***Bicycle2.sfx*** in the File Name text box (see Figure 4-12).

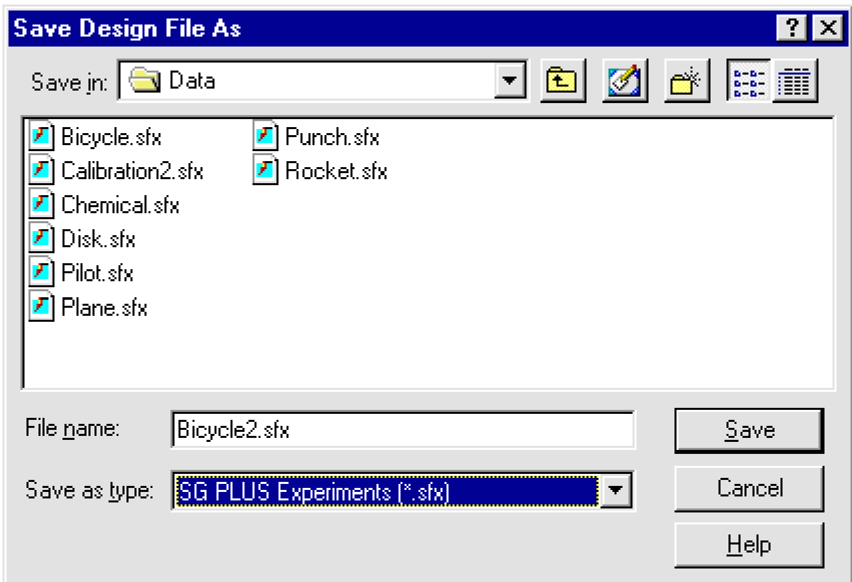


Figure 4-12. Completed Save Design File As Dialog Box

3. Click Save to save the augmented design.

Entering a Second Set of Experimental Results

So far, the conclusions have been tentative. To investigate further, you will add another eight runs to the augmented design and reanalyze it.

Gear has had a large effect but was confounded with two-factor interactions. You will add the eight runs to clear the **Gear** variable of all confounding and determine if it really has a significant effect on **Time**.

1. Choose WINDOW... BICYCLE2.SFX... from the Menu bar...

2. Tab to the **Time** column and down to Row 9.

Notice that Rows 9 through 16 are empty (see Figure 4-13).

Bicycle2.sfx

	gear	raincoat	breakfast	tires	time
1	high	off	no	hard	69
2	low	on	no	soft	52
3	low	off	yes	soft	60
4	high	on	yes	hard	83
5	high	on	yes	soft	71
6	low	off	yes	hard	50
7	low	on	no	hard	59
8	high	off	no	soft	88
9	low	off	no	hard	
10	high	on	no	soft	
11	high	off	yes	soft	
12	low	on	yes	hard	
13	low	on	yes	soft	
14	high	off	yes	hard	

Figure 4-13. The DataSheet Showing the Blank Rows in the Time Column

3. Enter the following values to the remaining rows in the column: **53**, **78**, **87**, **60**, **47**, **74**, **84**, **62** (see Figure 4-14).

Bicycle2.sfx

	gear	raincoat	breakfast	tires	time
4	high	on	yes	hard	83
5	high	on	yes	soft	71
6	low	off	yes	hard	50
7	low	on	no	hard	59
8	high	off	no	soft	88
9	low	off	no	hard	53
10	high	on	no	soft	78
11	high	off	yes	soft	87
12	low	on	yes	hard	60
13	low	on	yes	soft	47
14	high	off	yes	hard	74
15	high	on	no	hard	84
16	low	off	no	soft	62
17					

Figure 4-14. Completed DataSheet with Added Rows

4. Choose FILE... SAVE... SAVE DESIGN FILE... to save the completed design file.

Analyzing the Augmented Design

1. Choose SPECIAL... EXPERIMENTAL DESIGN... ANALYZE DESIGN... to display the Analyze Design dialog box.

2. Enter **Time** into the Data text box and click OK to display the Analysis Summary and Pareto Chart in the Analysis window (see Figure 4-15).

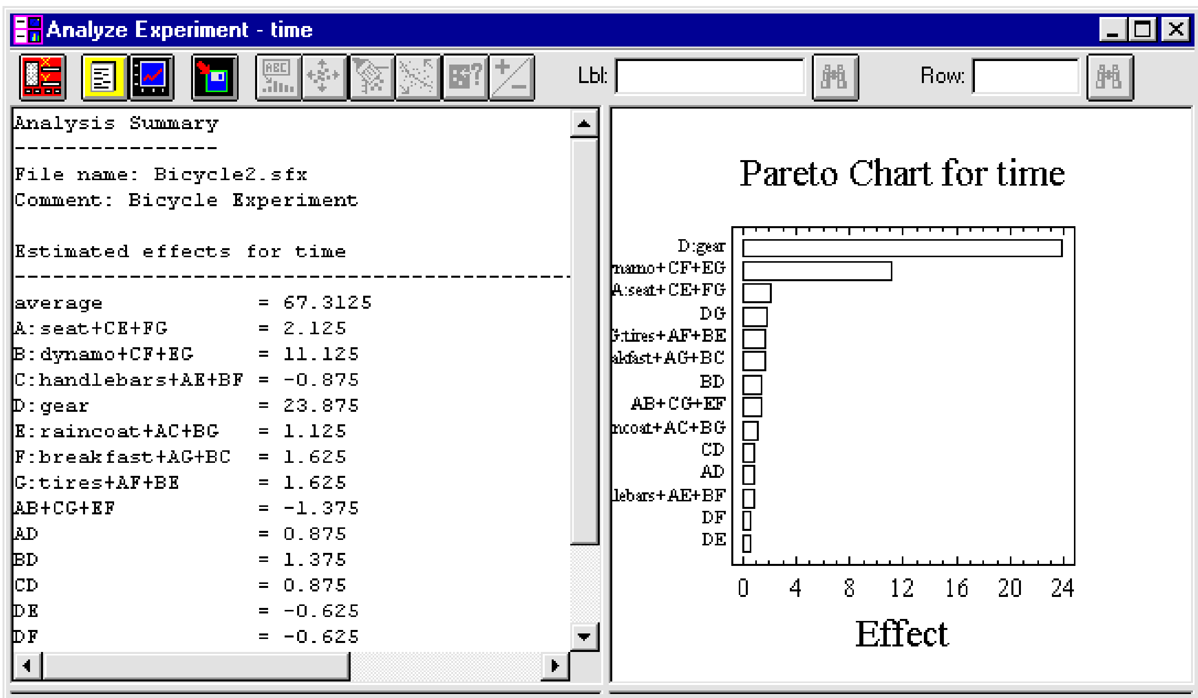


Figure 4-15. Analysis Summary and Pareto Chart in the Analysis Window

3. Maximize the Analysis Summary (see Figure 4-16).

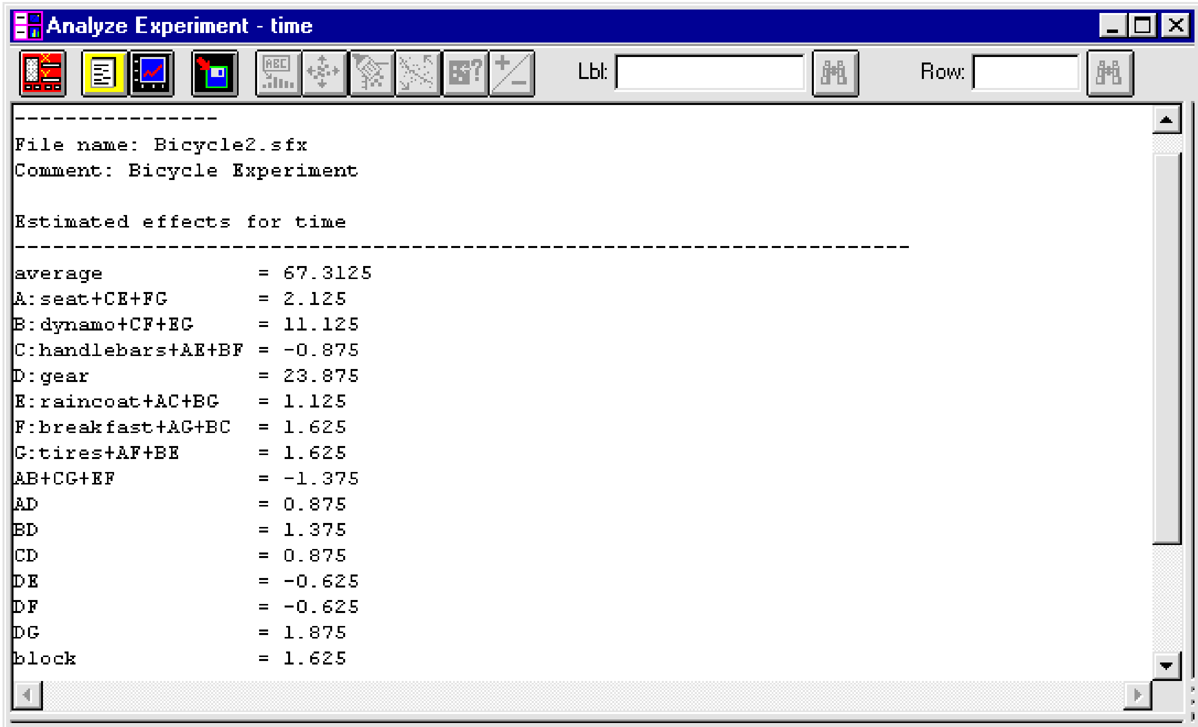


Figure 4-16. The Analysis Summary

The Analysis Summary shows that the **Gear** variable is no longer confounded and clearly has a large effect on **Time** (23.875 seconds).

References

Box, G. E. P., Hunter, W. G., and Hunter, J. S. 1978. *Statistics for Experimenters*. New York: John Wiley & Sons.